Kernelization Complexity of Solution Discovery Problems

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st-Vertex Cut Discovery Problem

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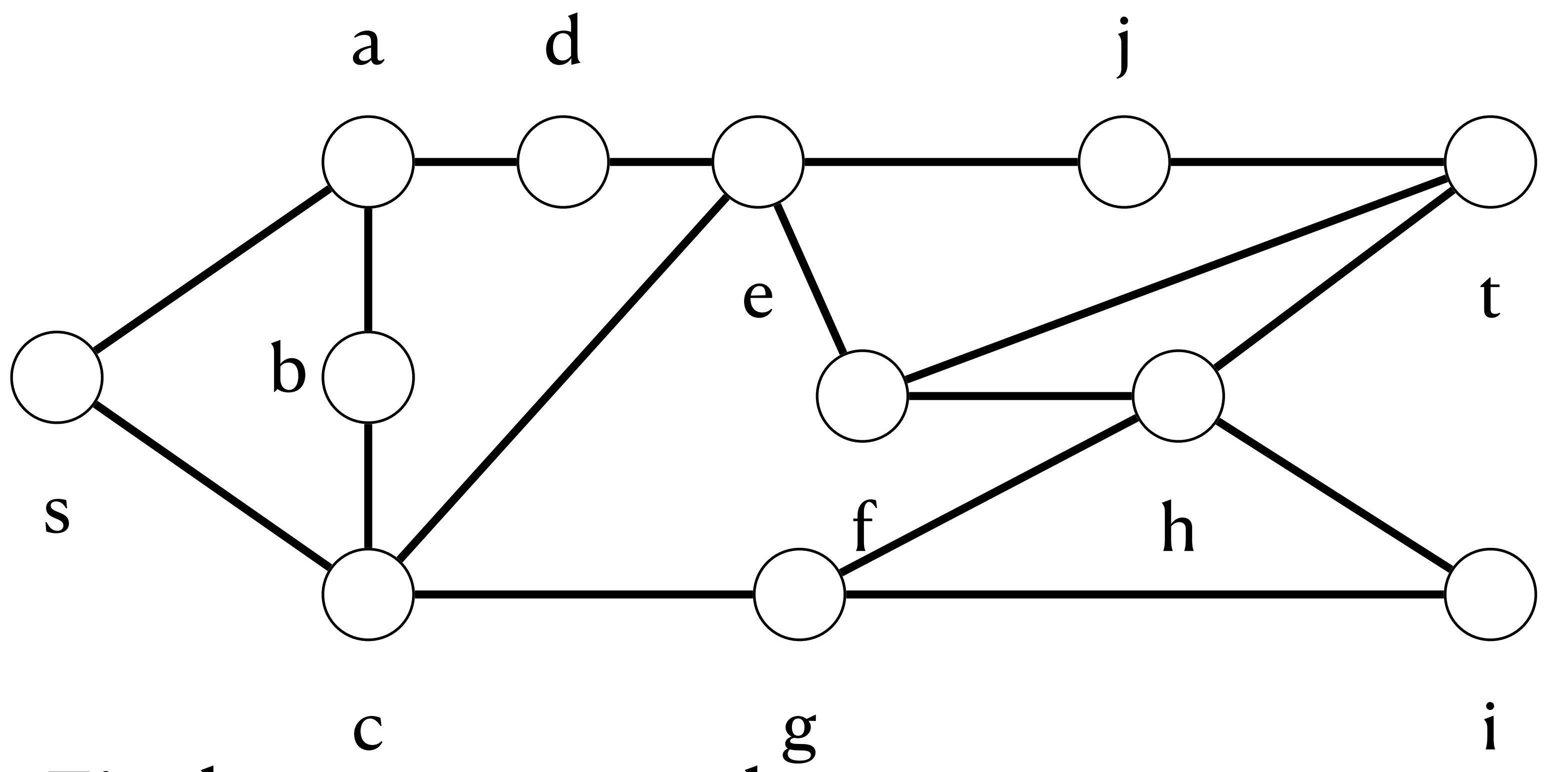

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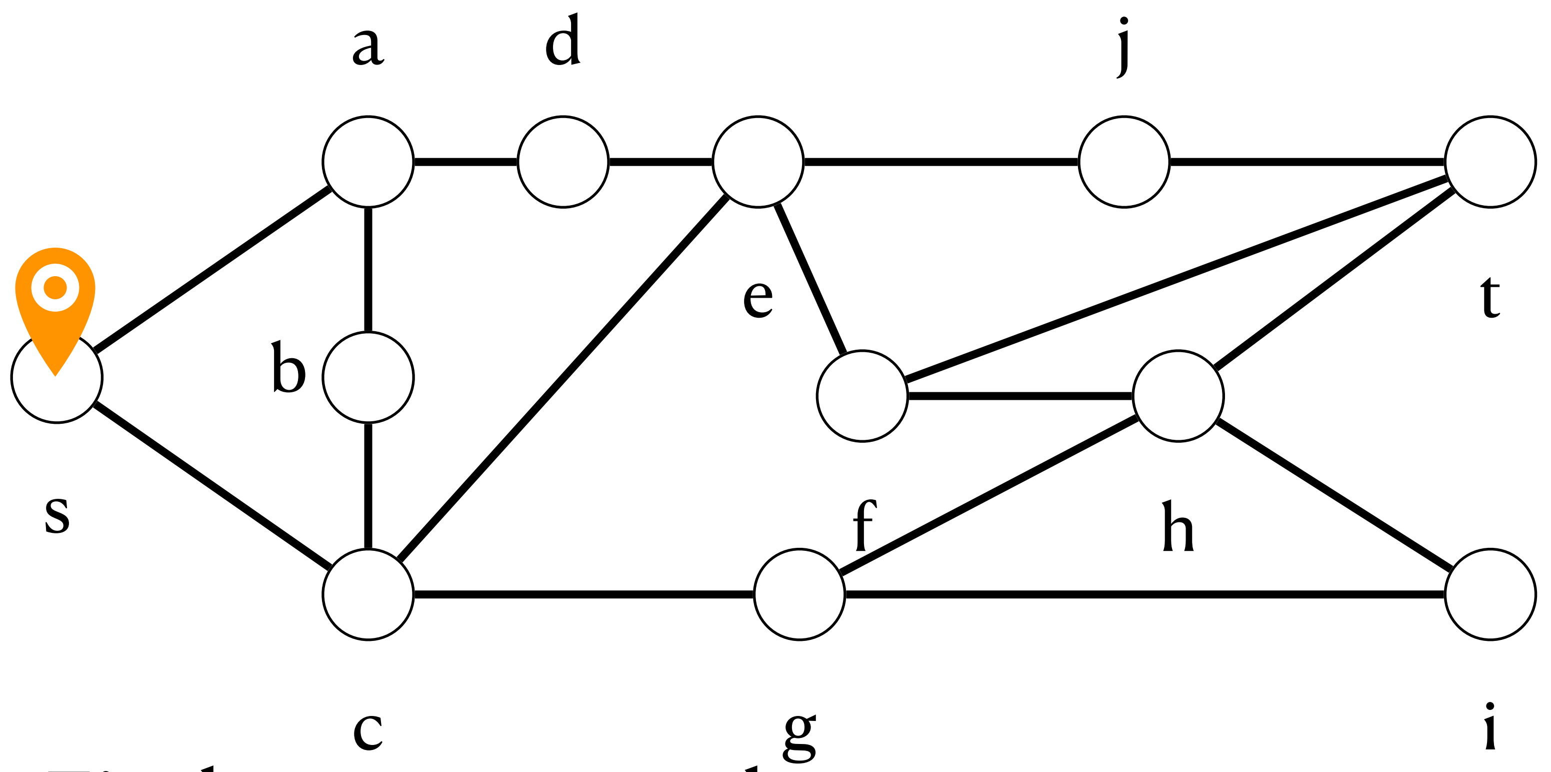

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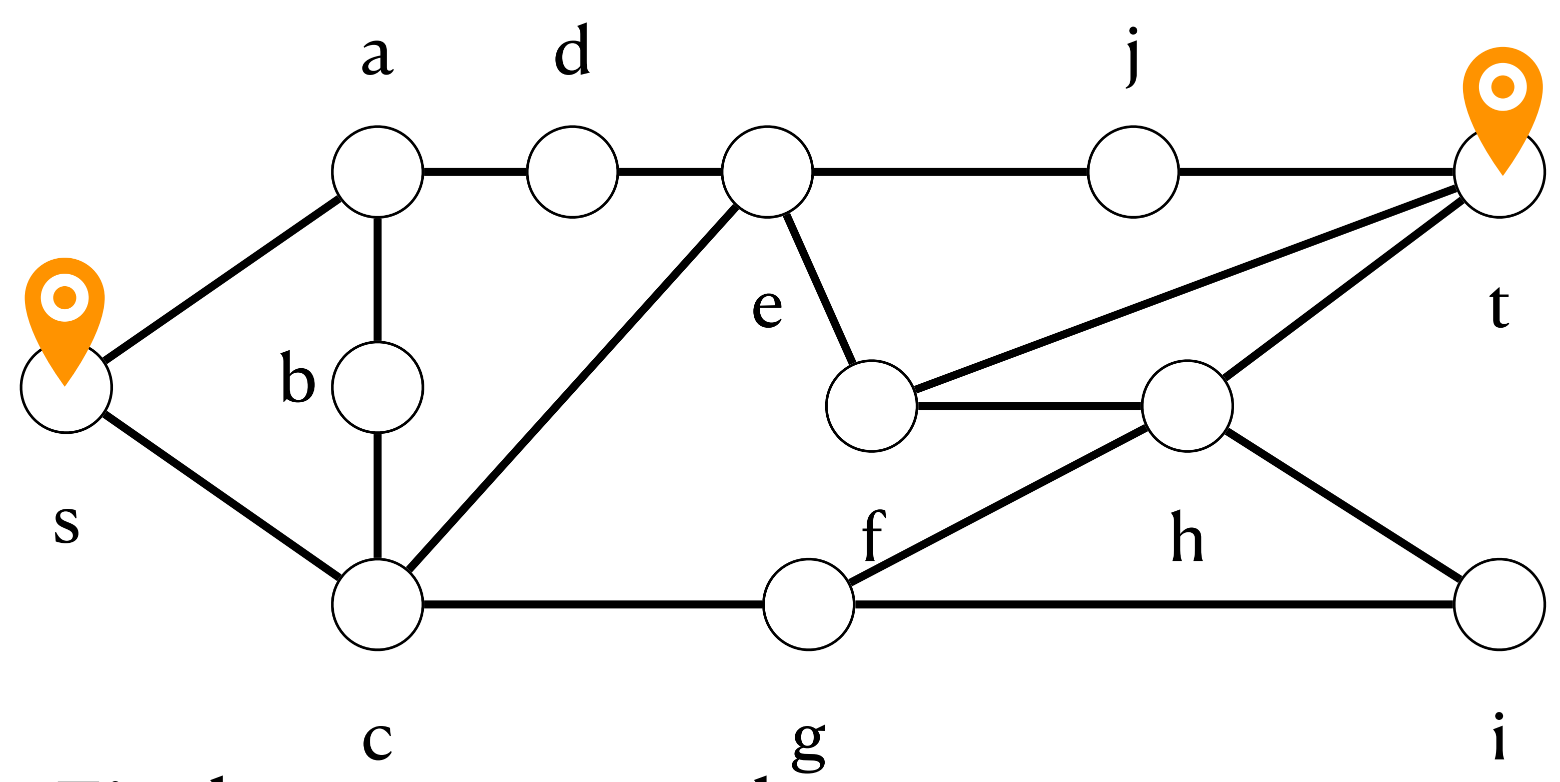

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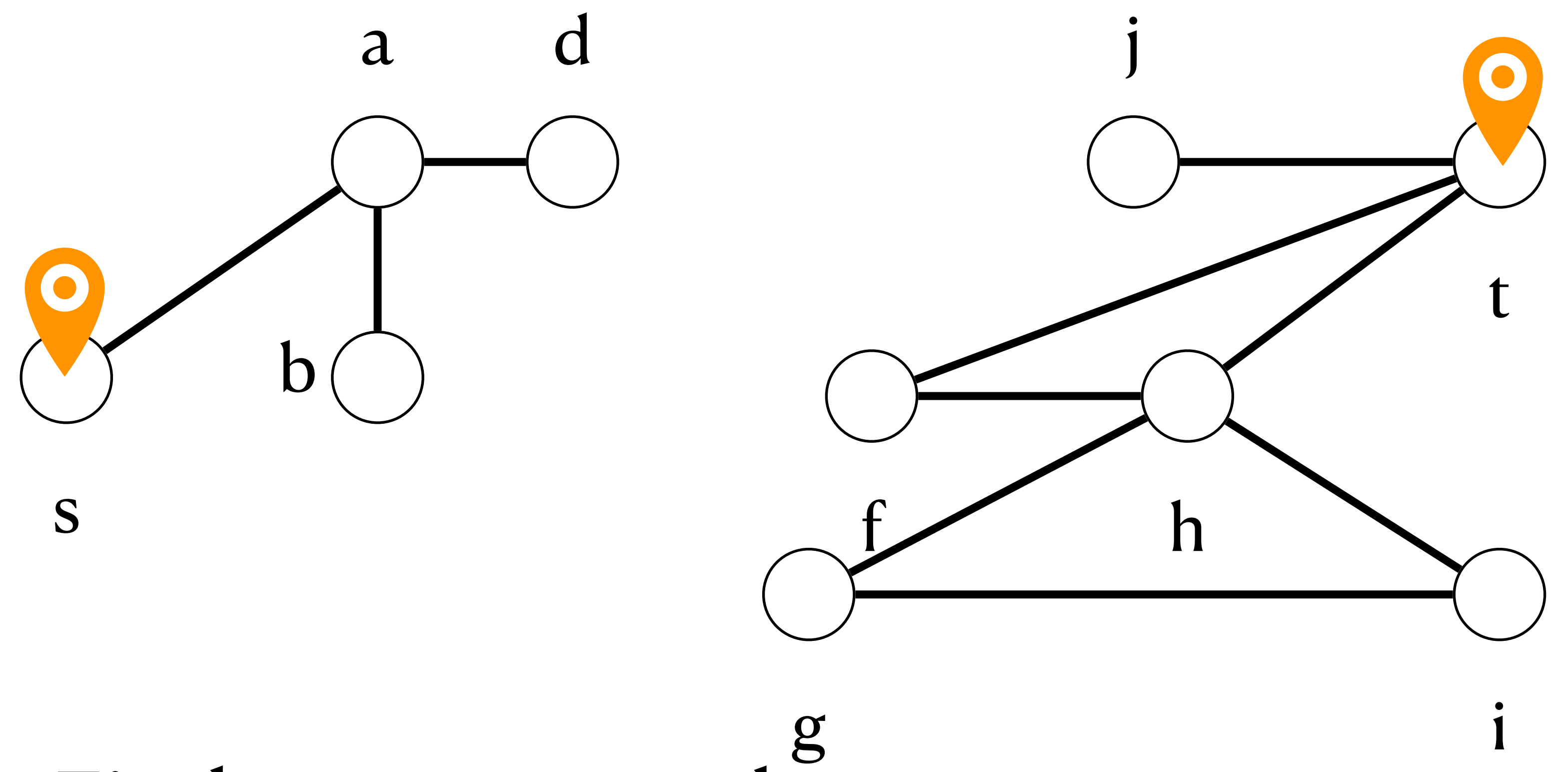

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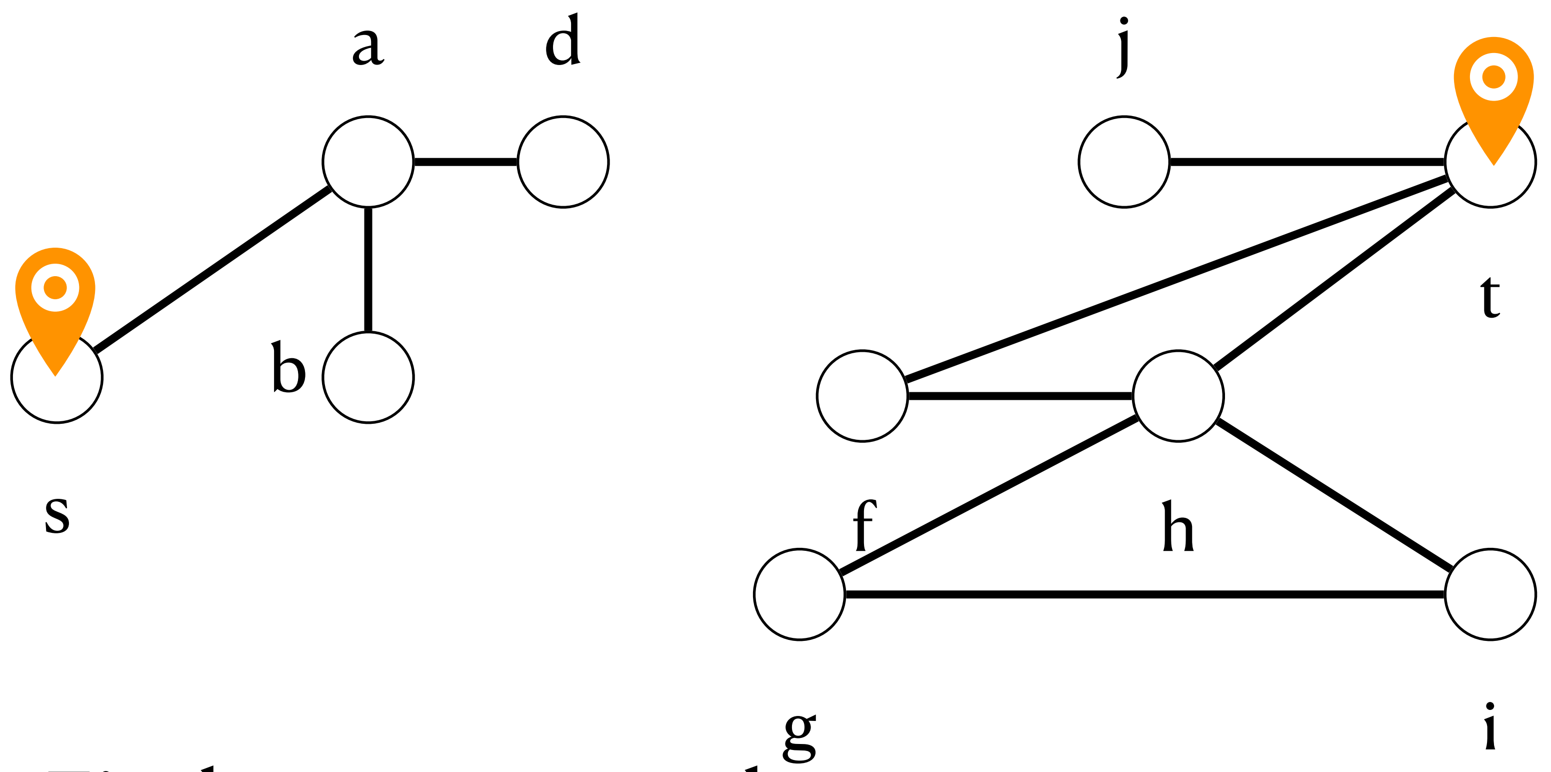

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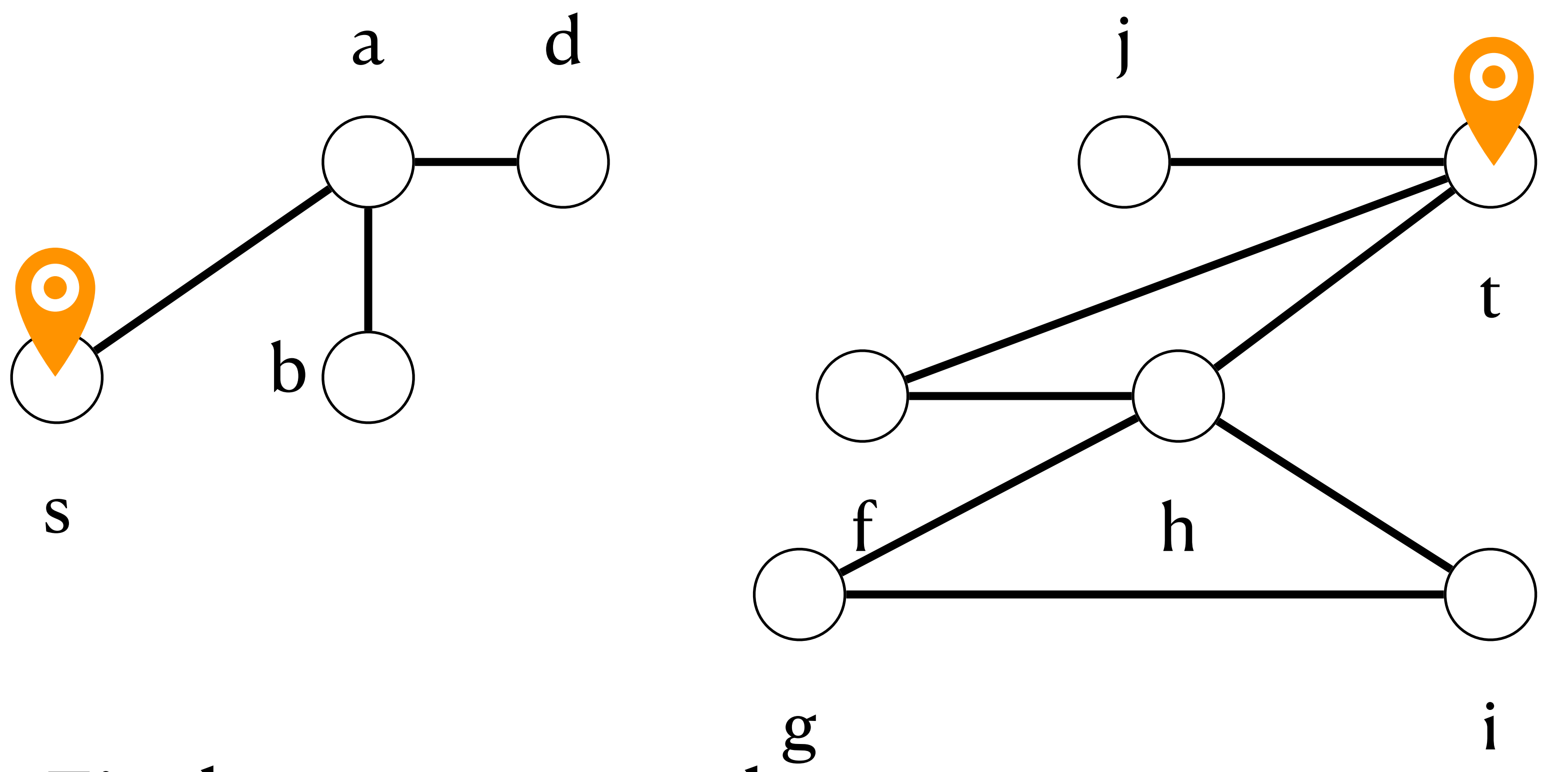

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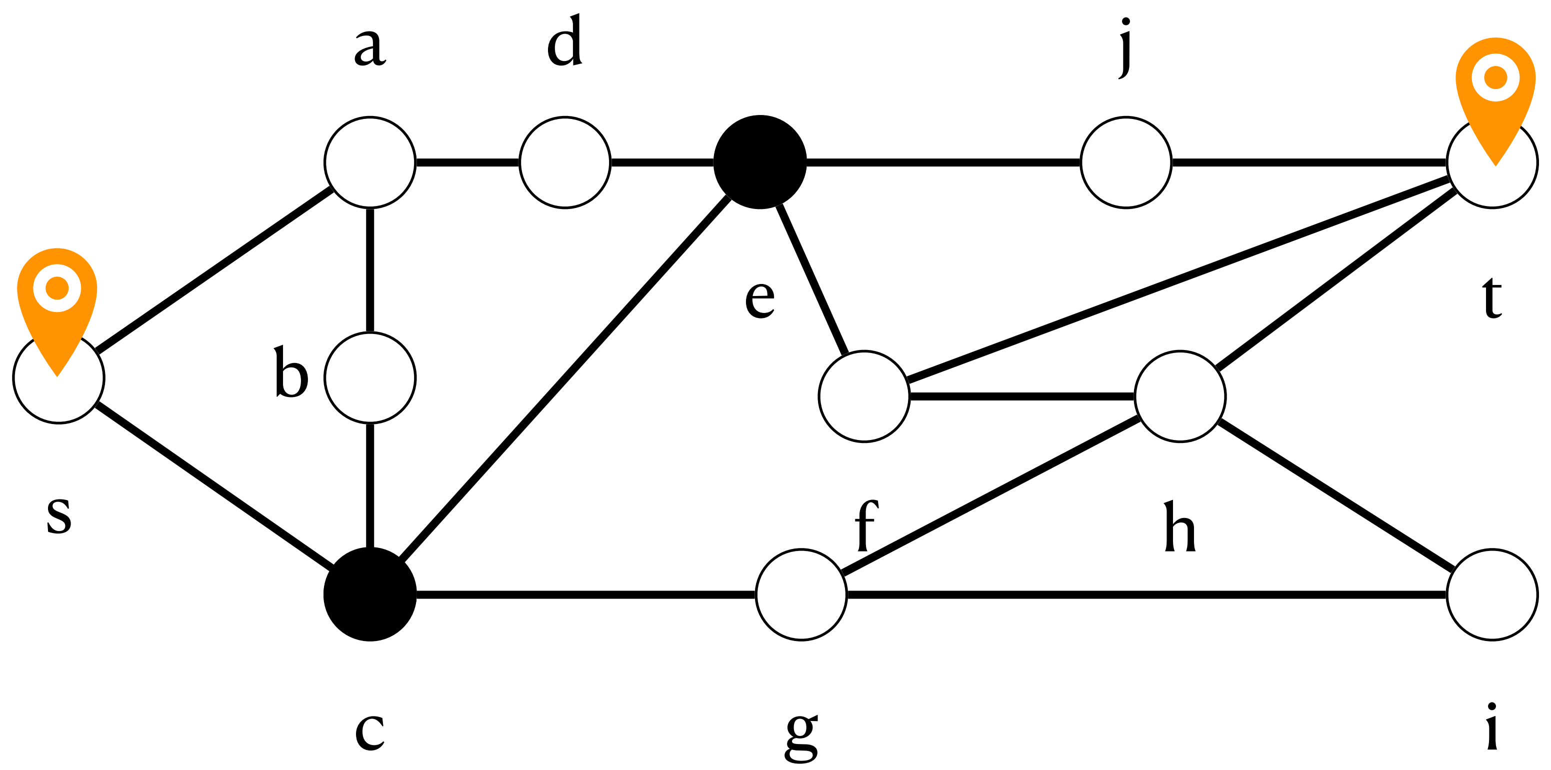

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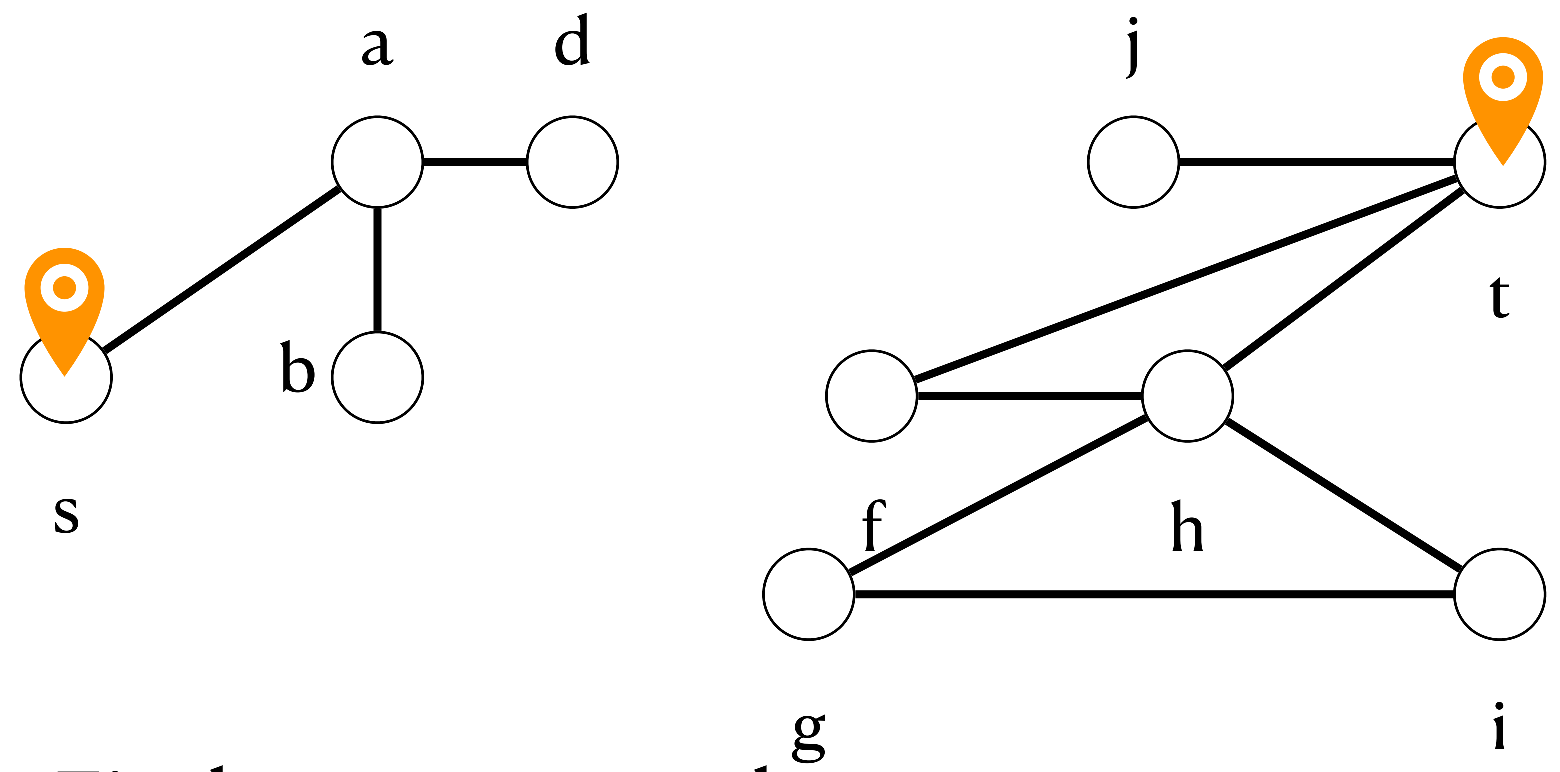

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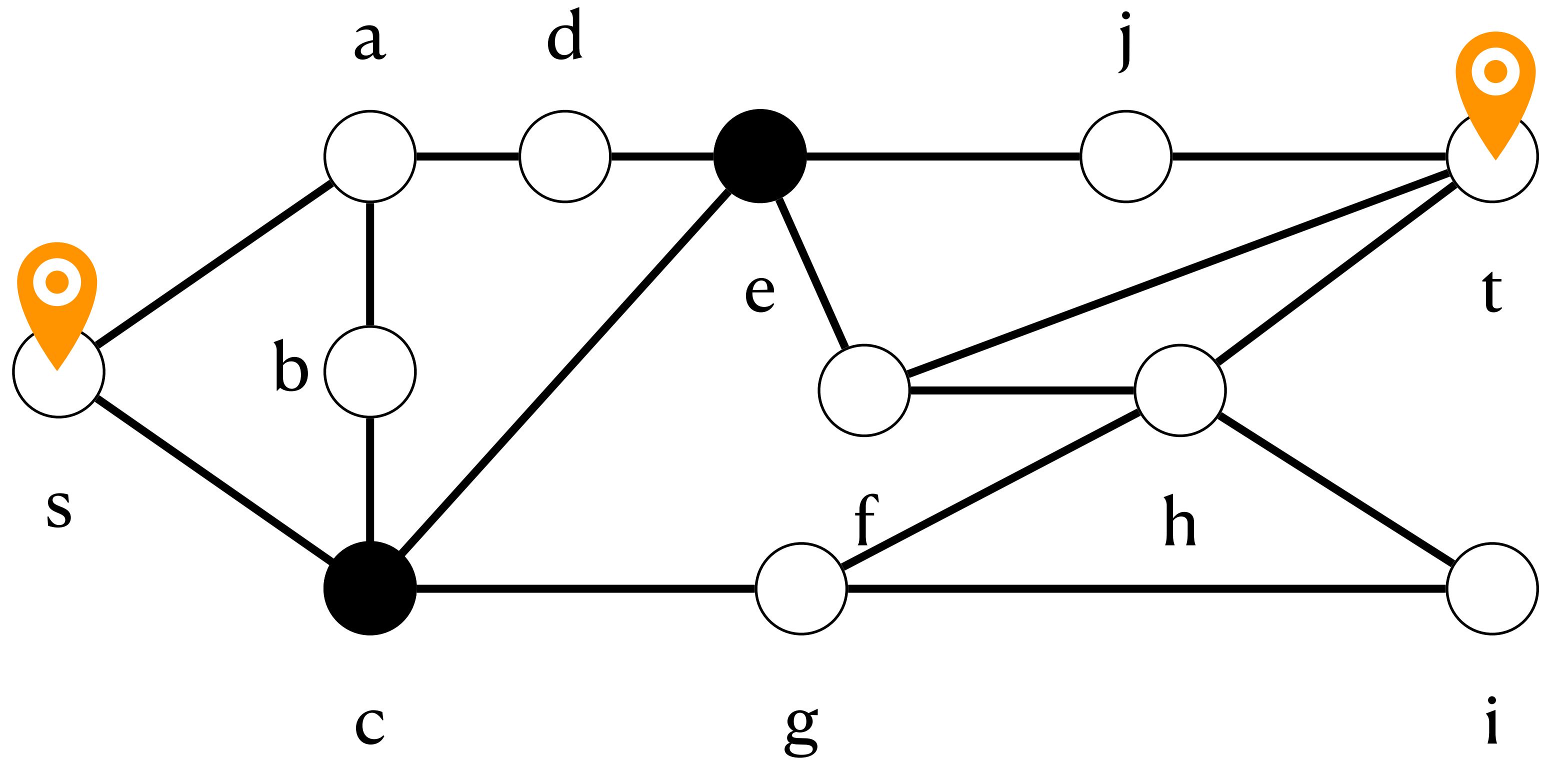

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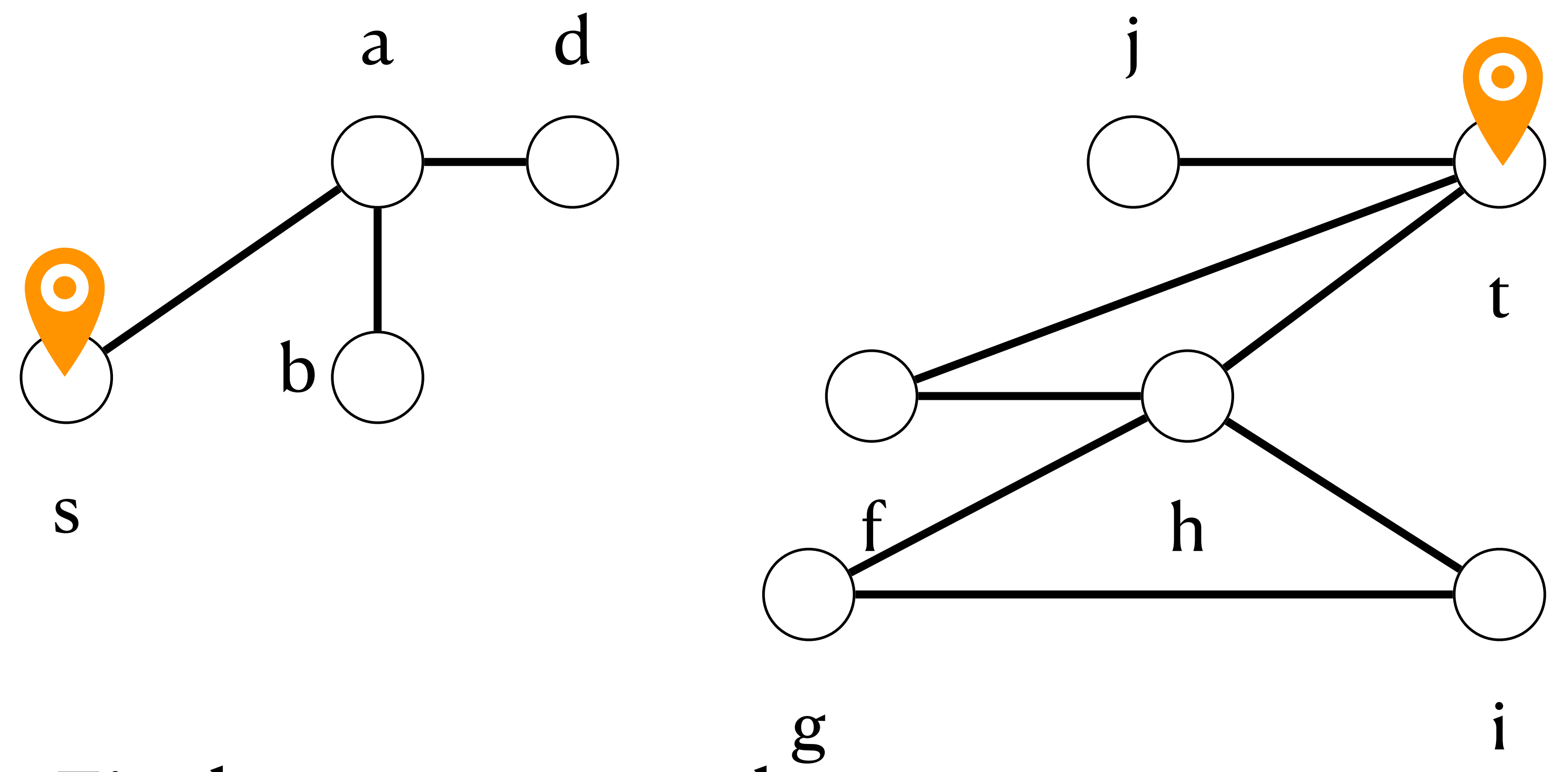

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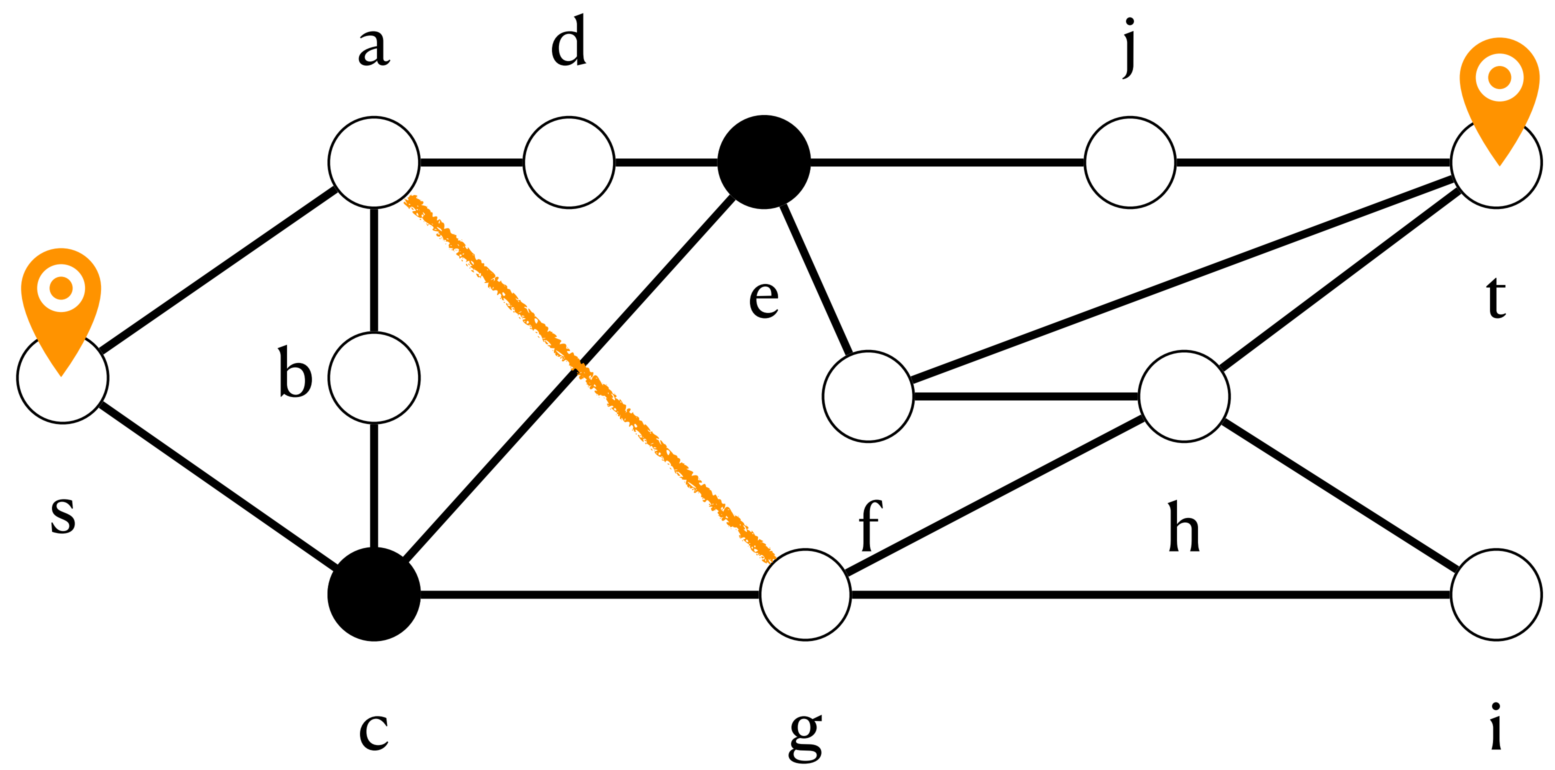

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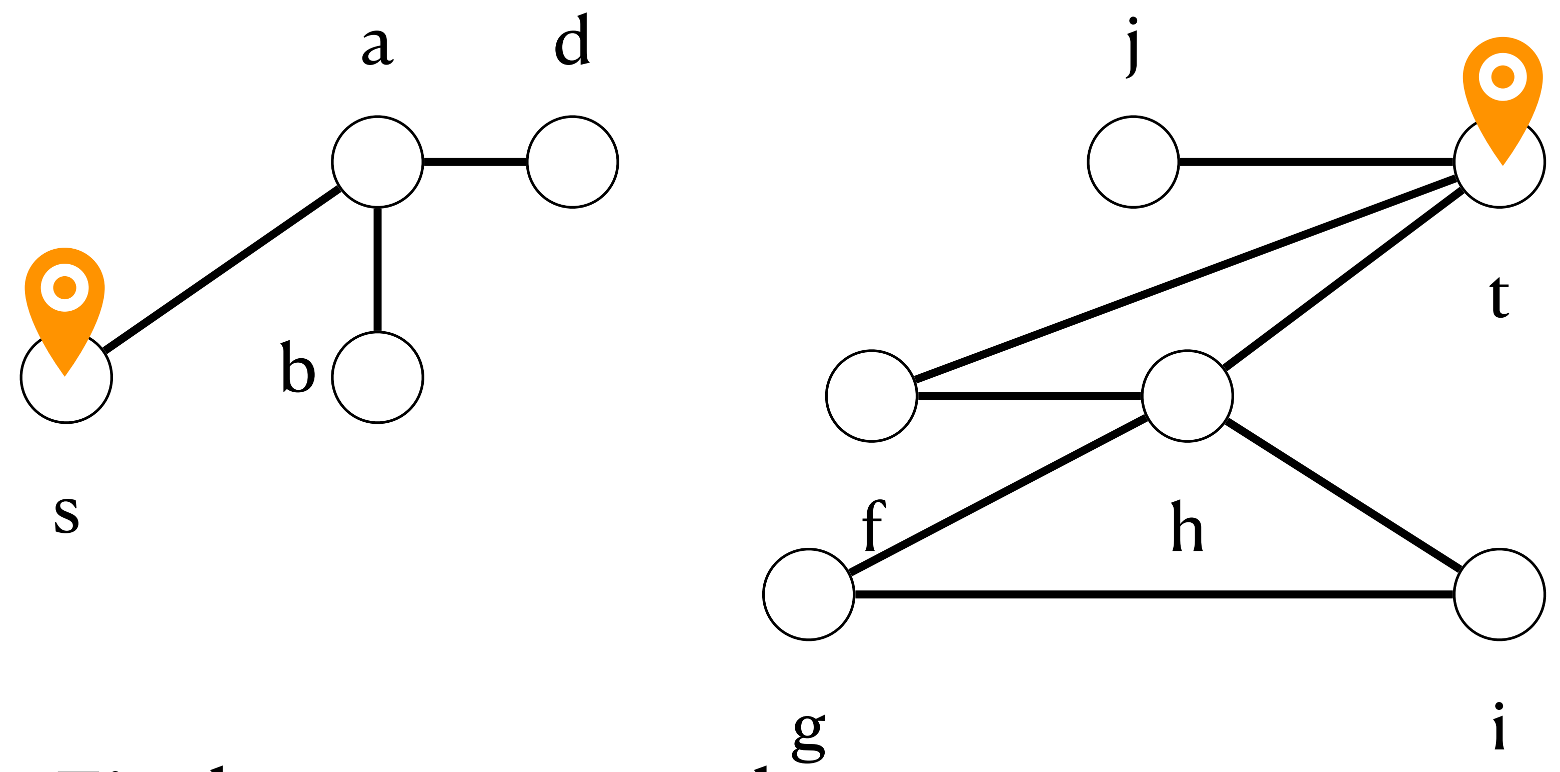

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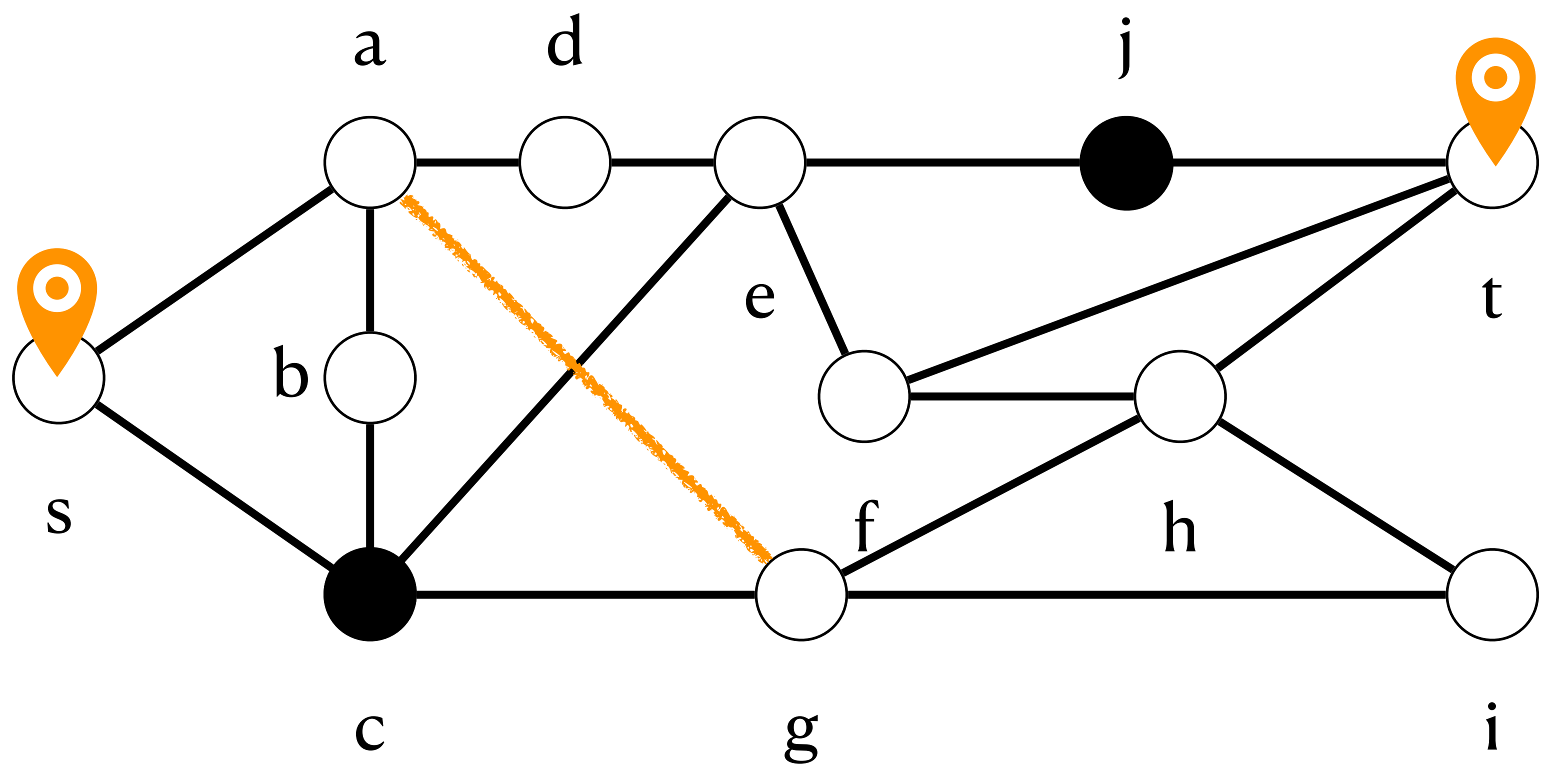

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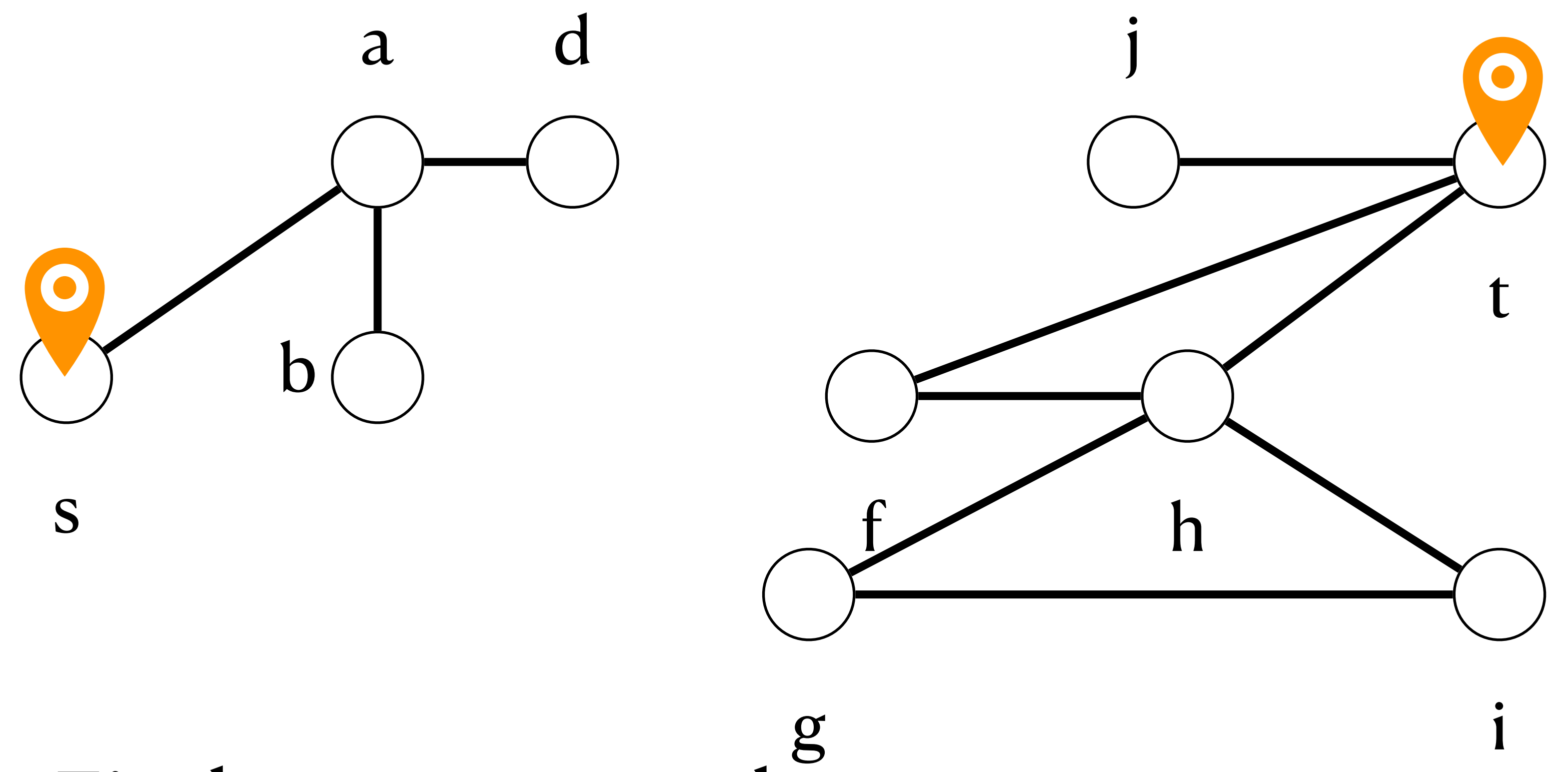

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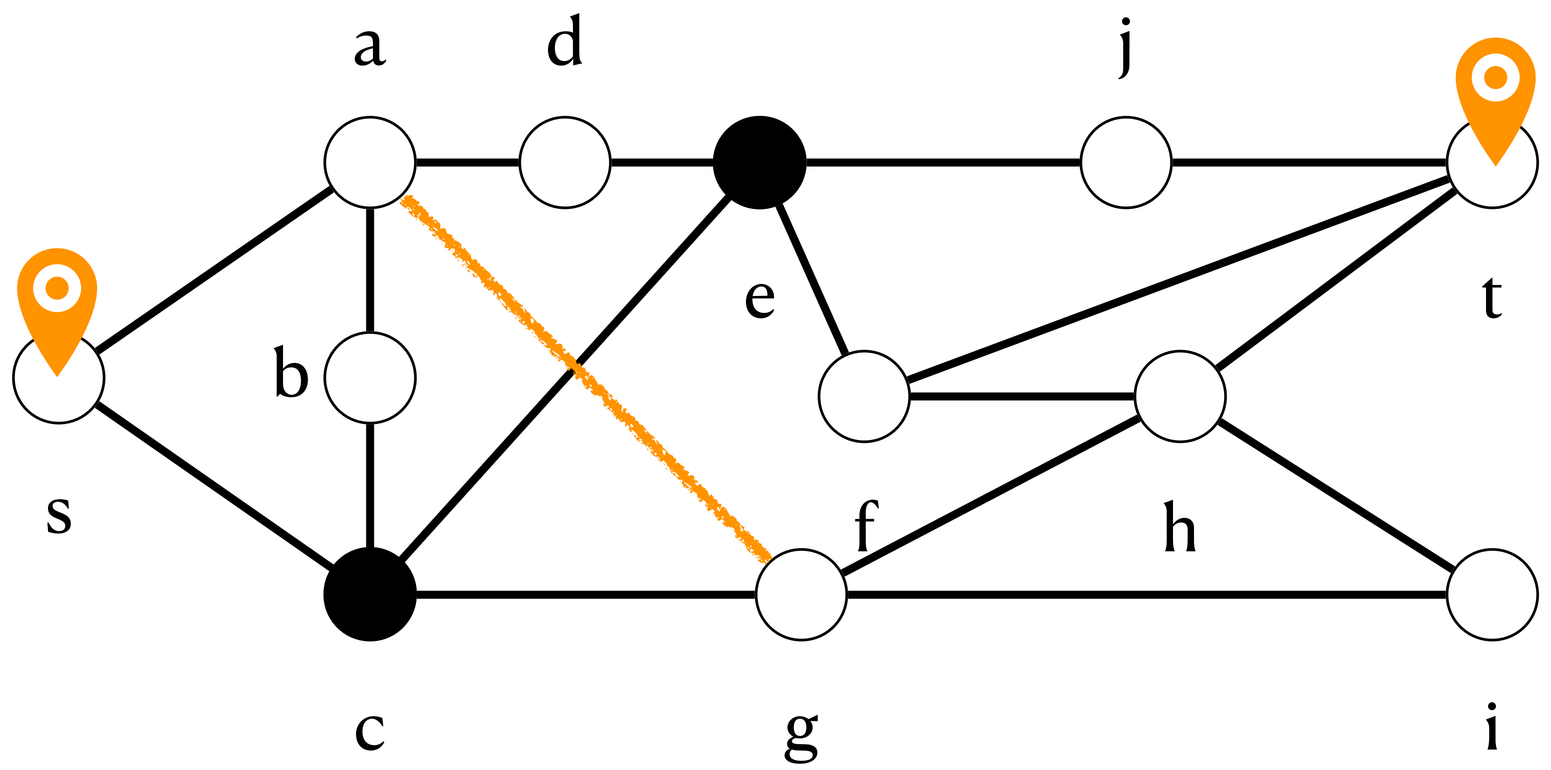

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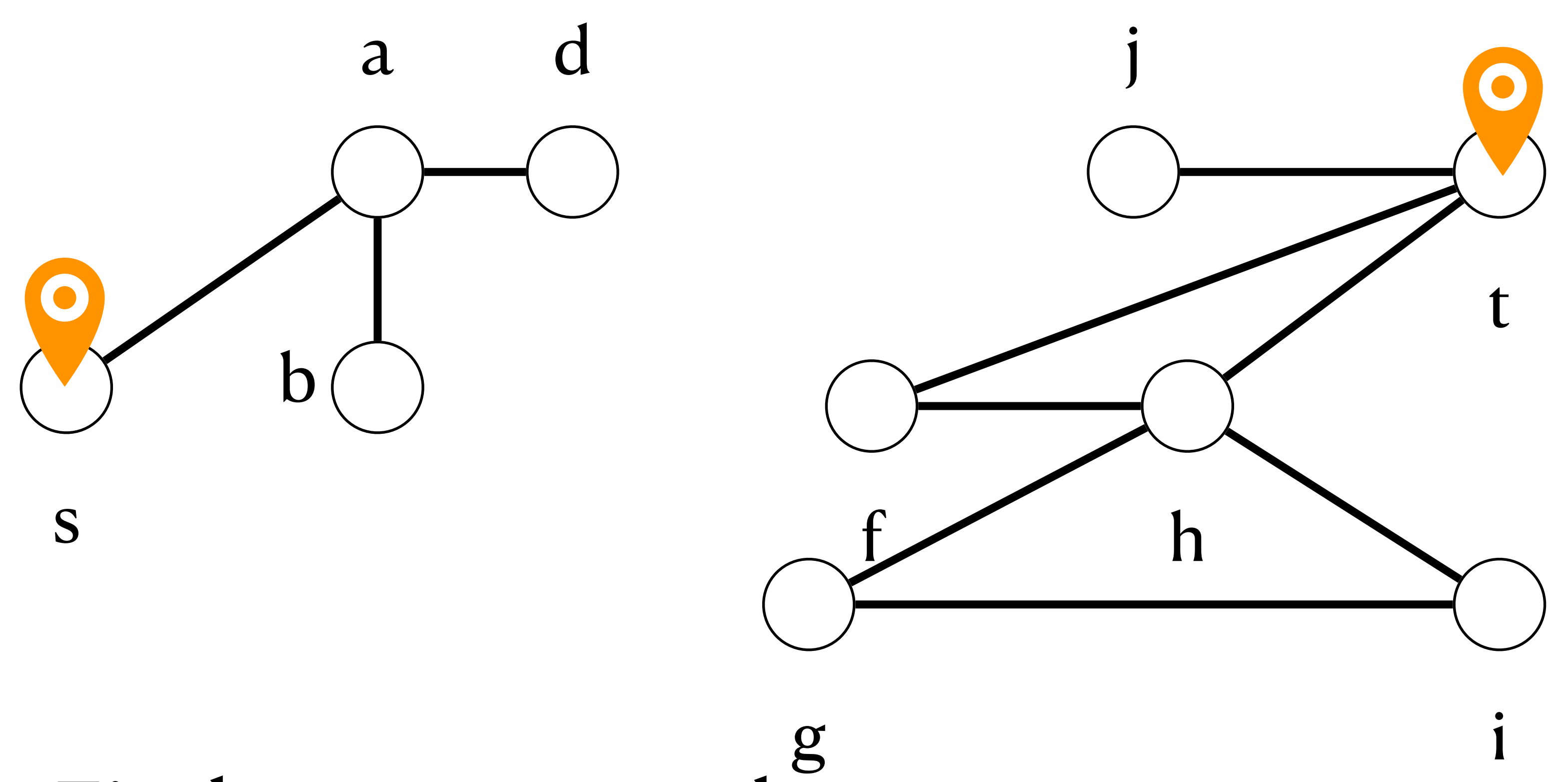

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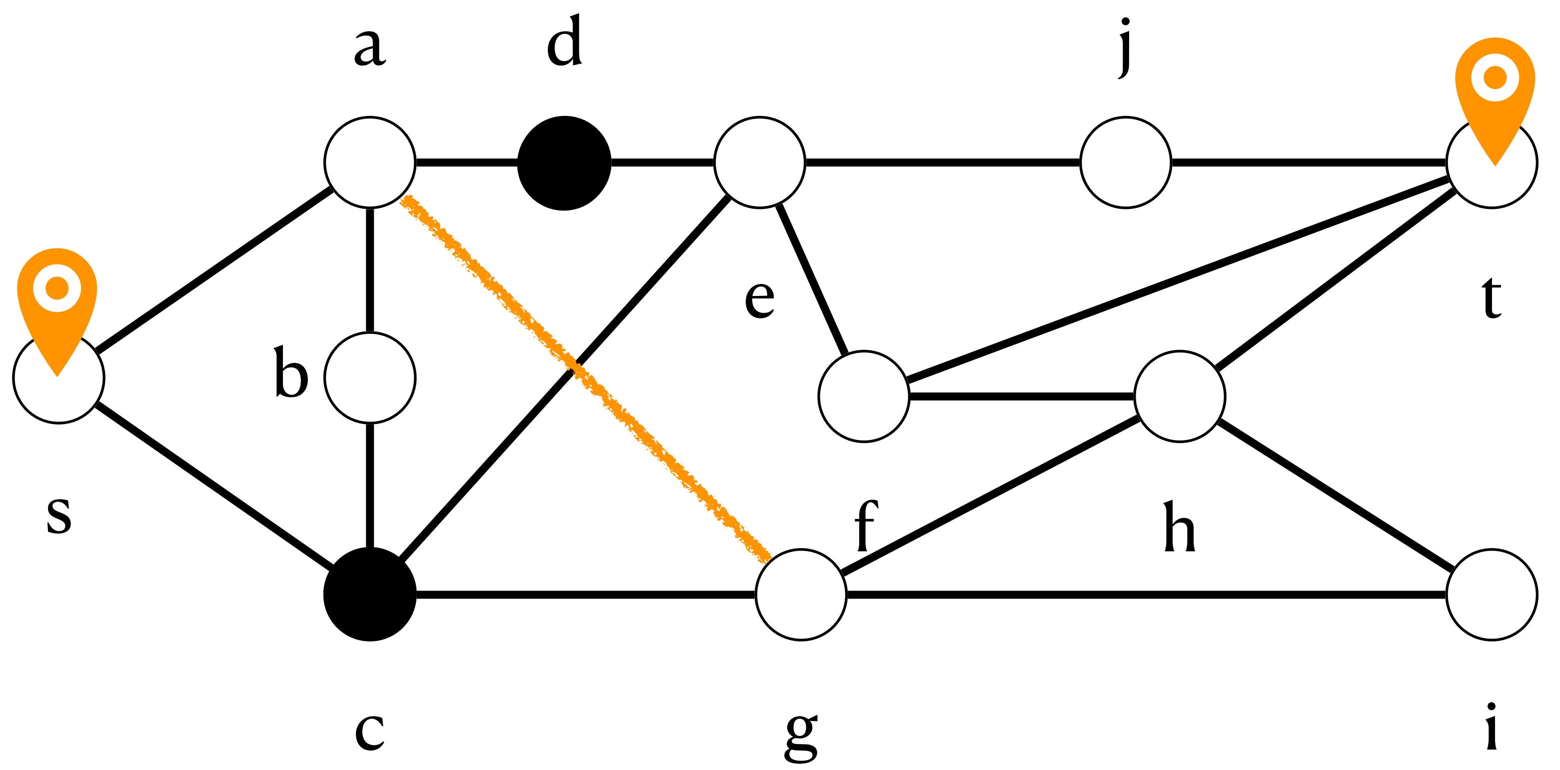

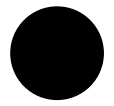 a resource, a token

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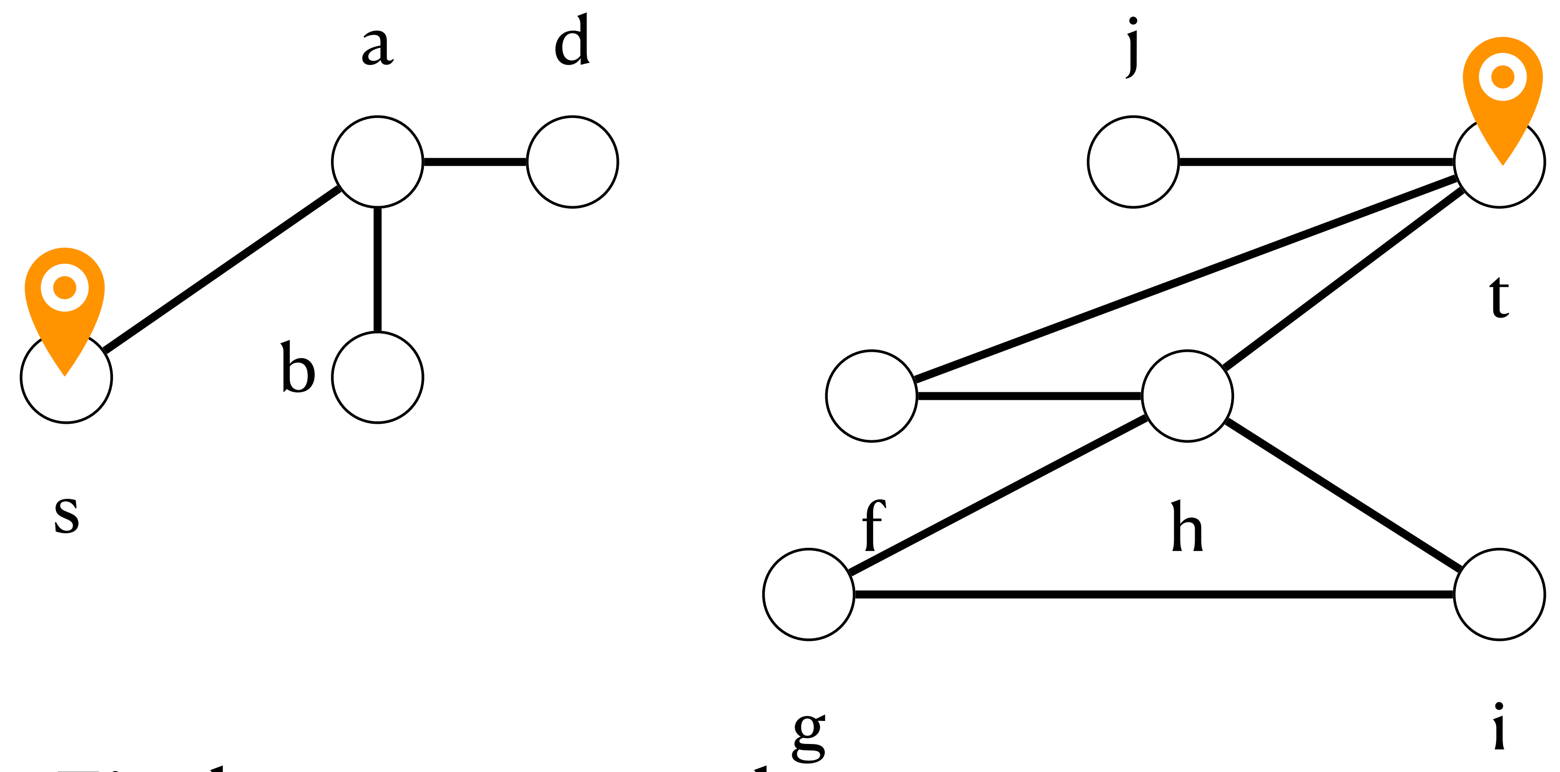

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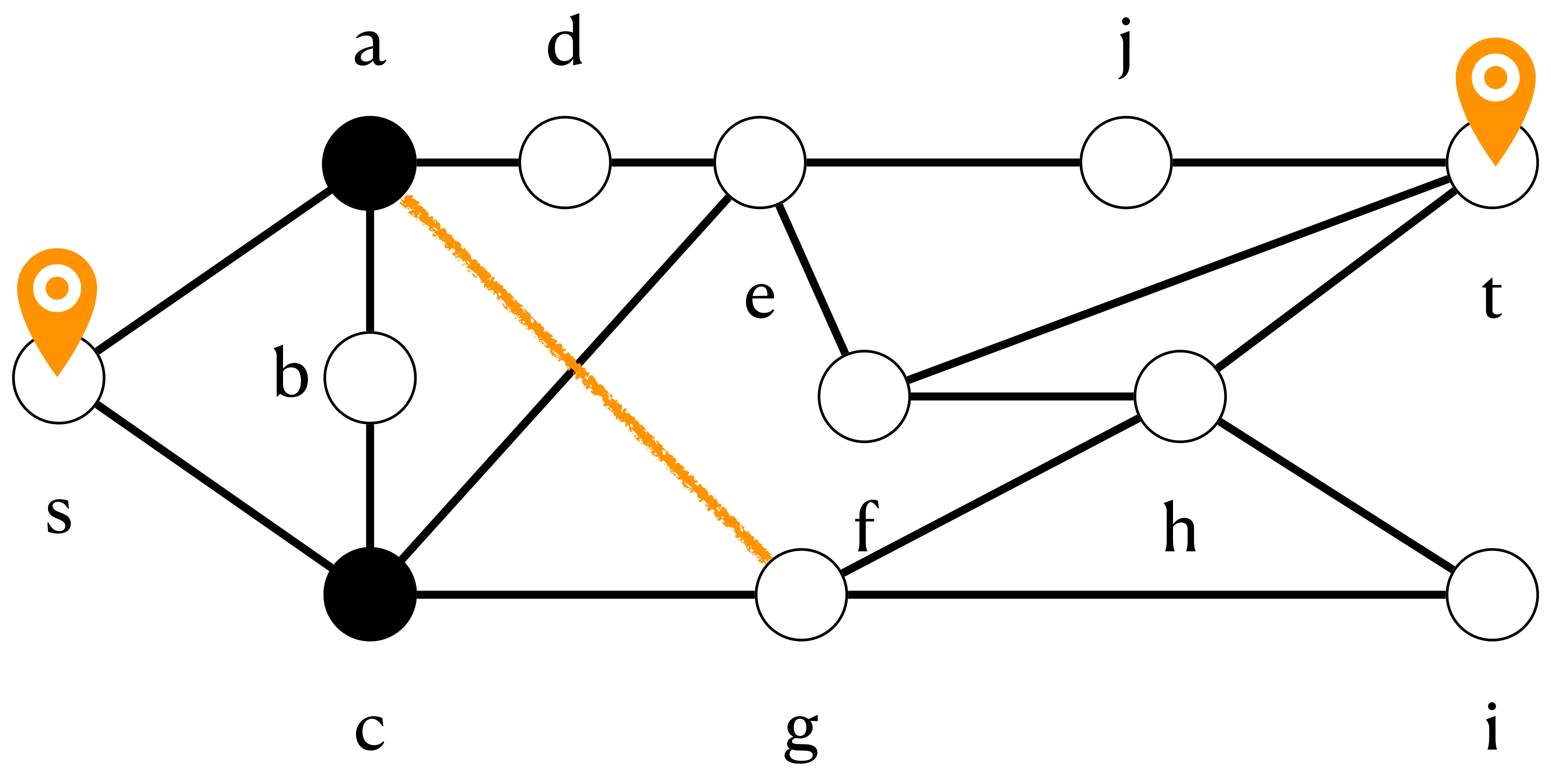

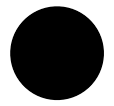 a resource, a token

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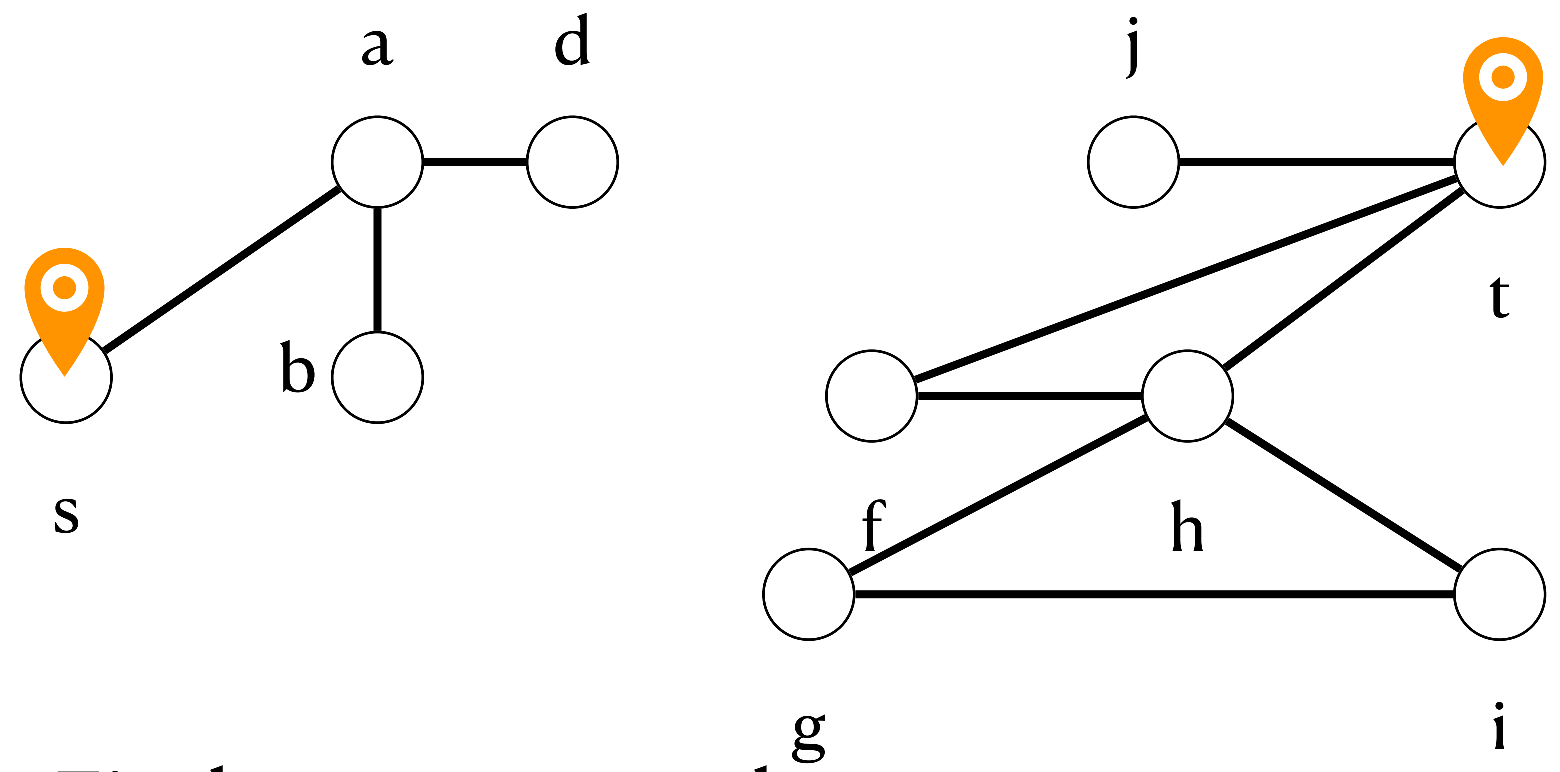

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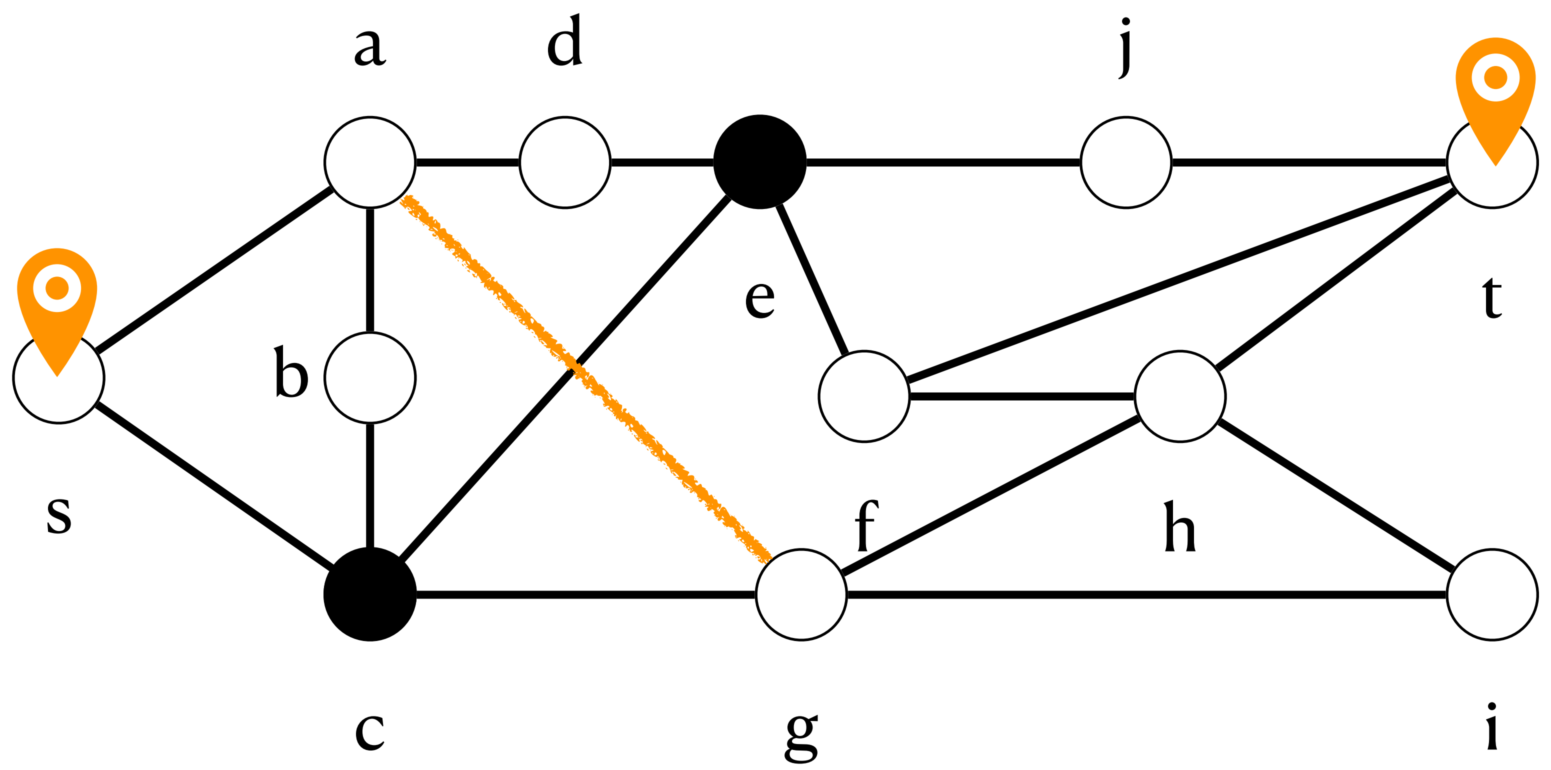

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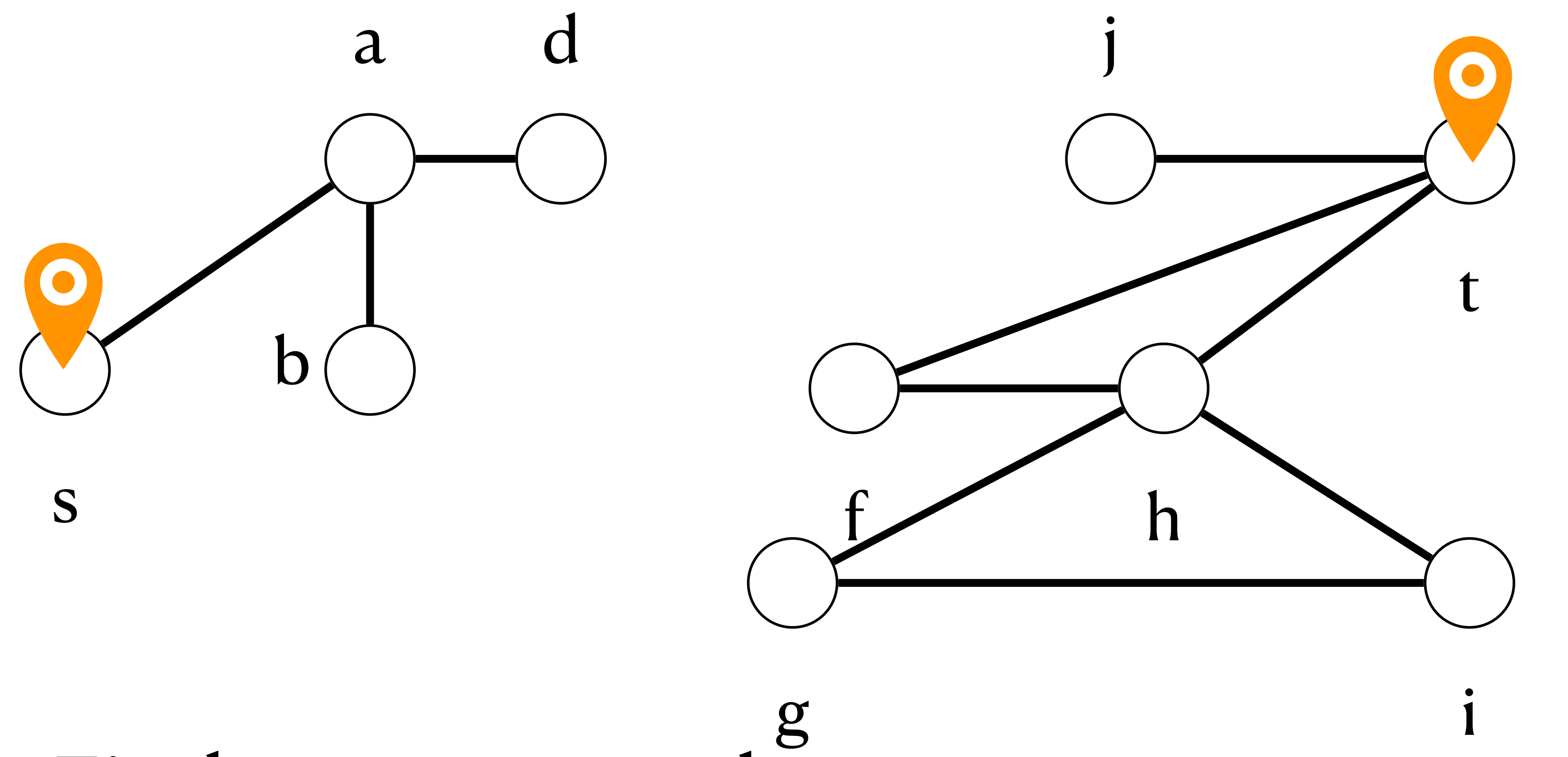

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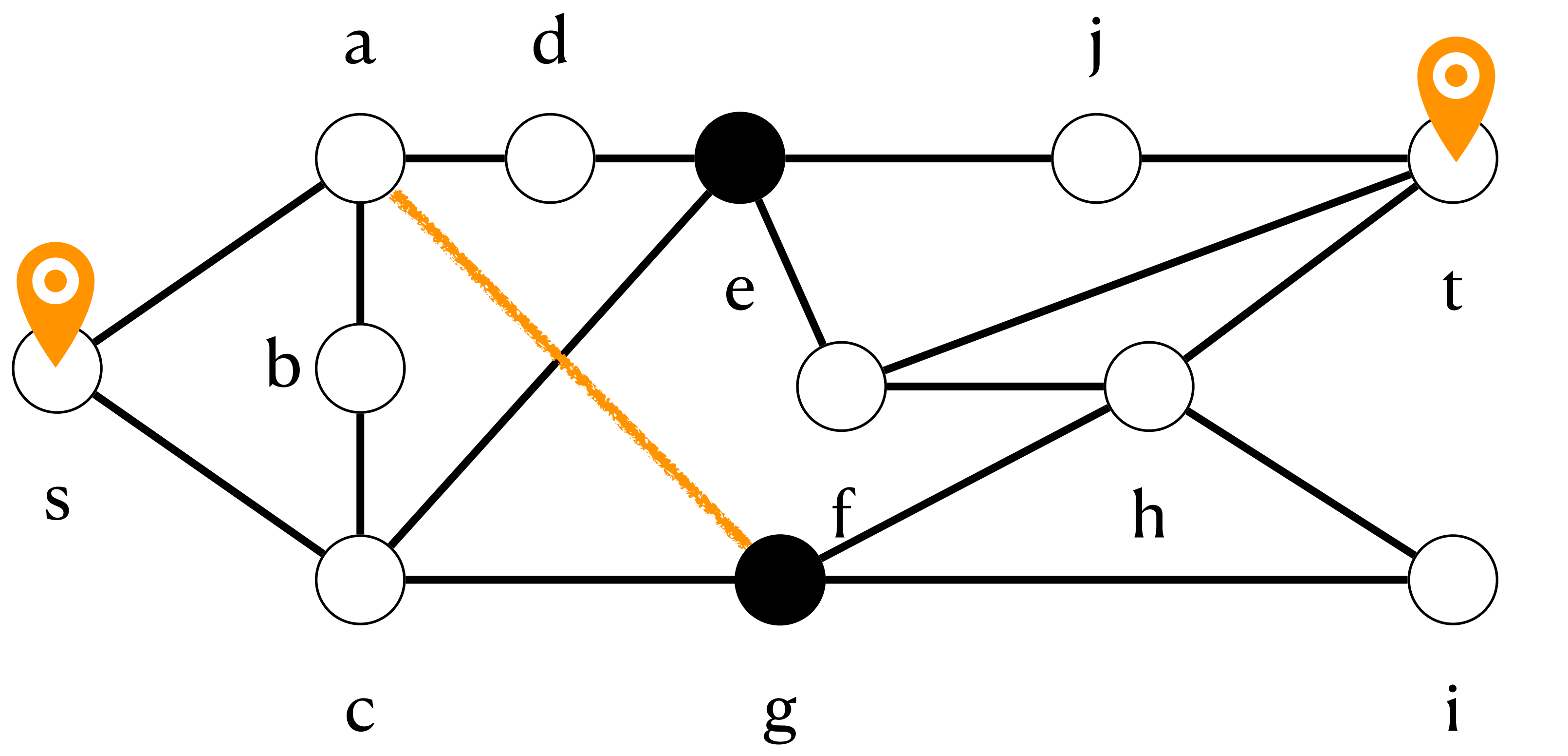

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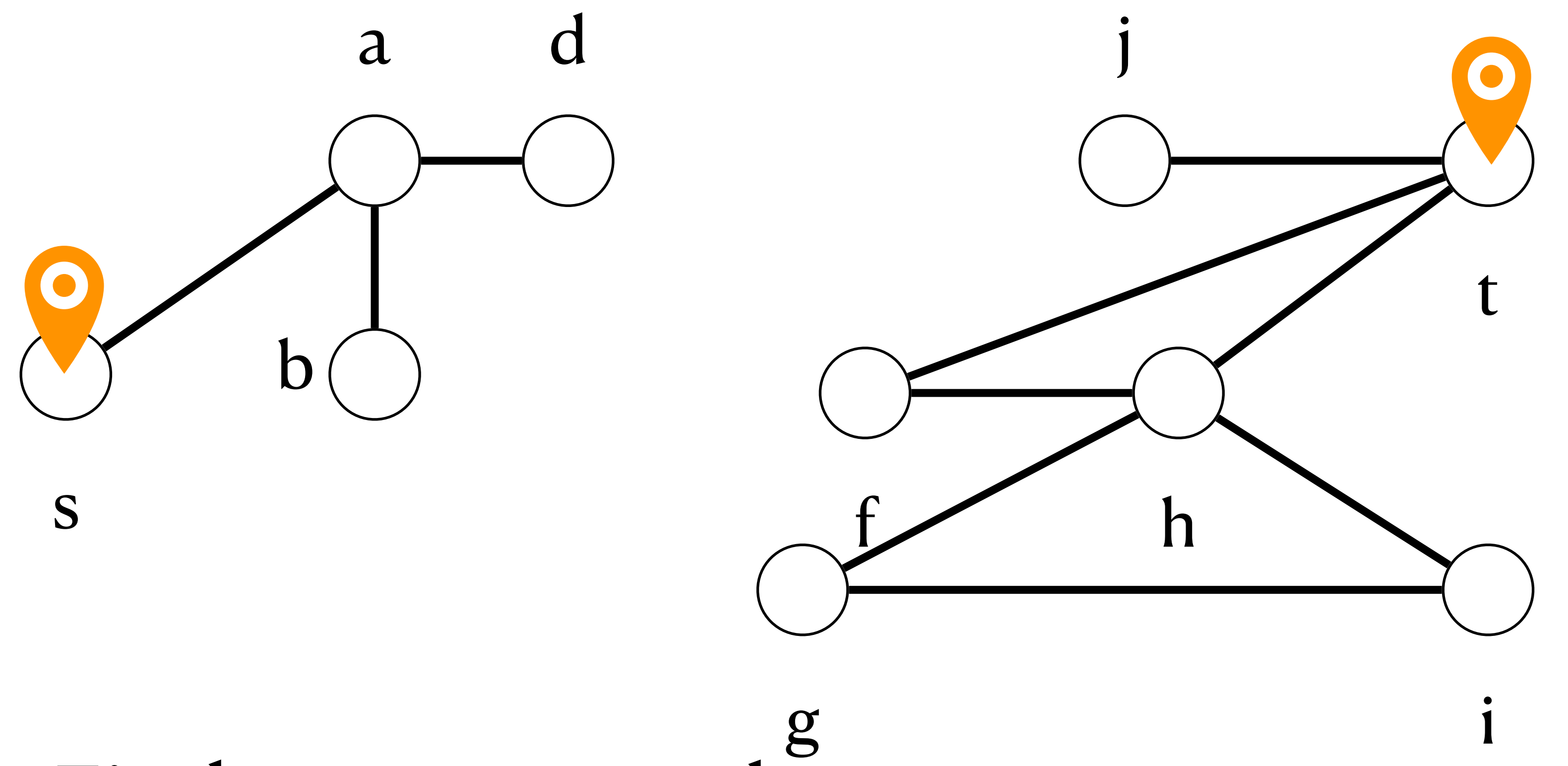

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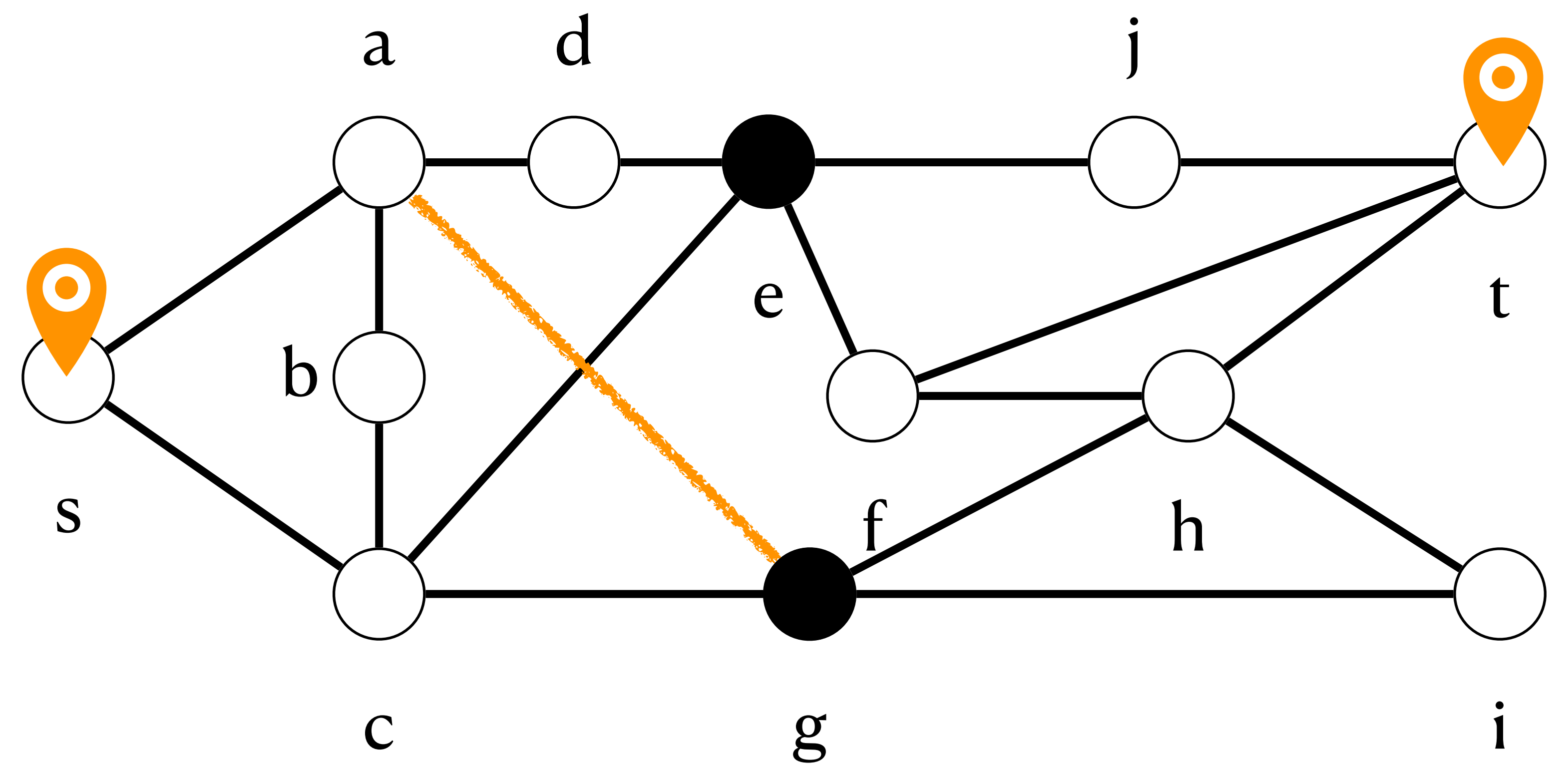

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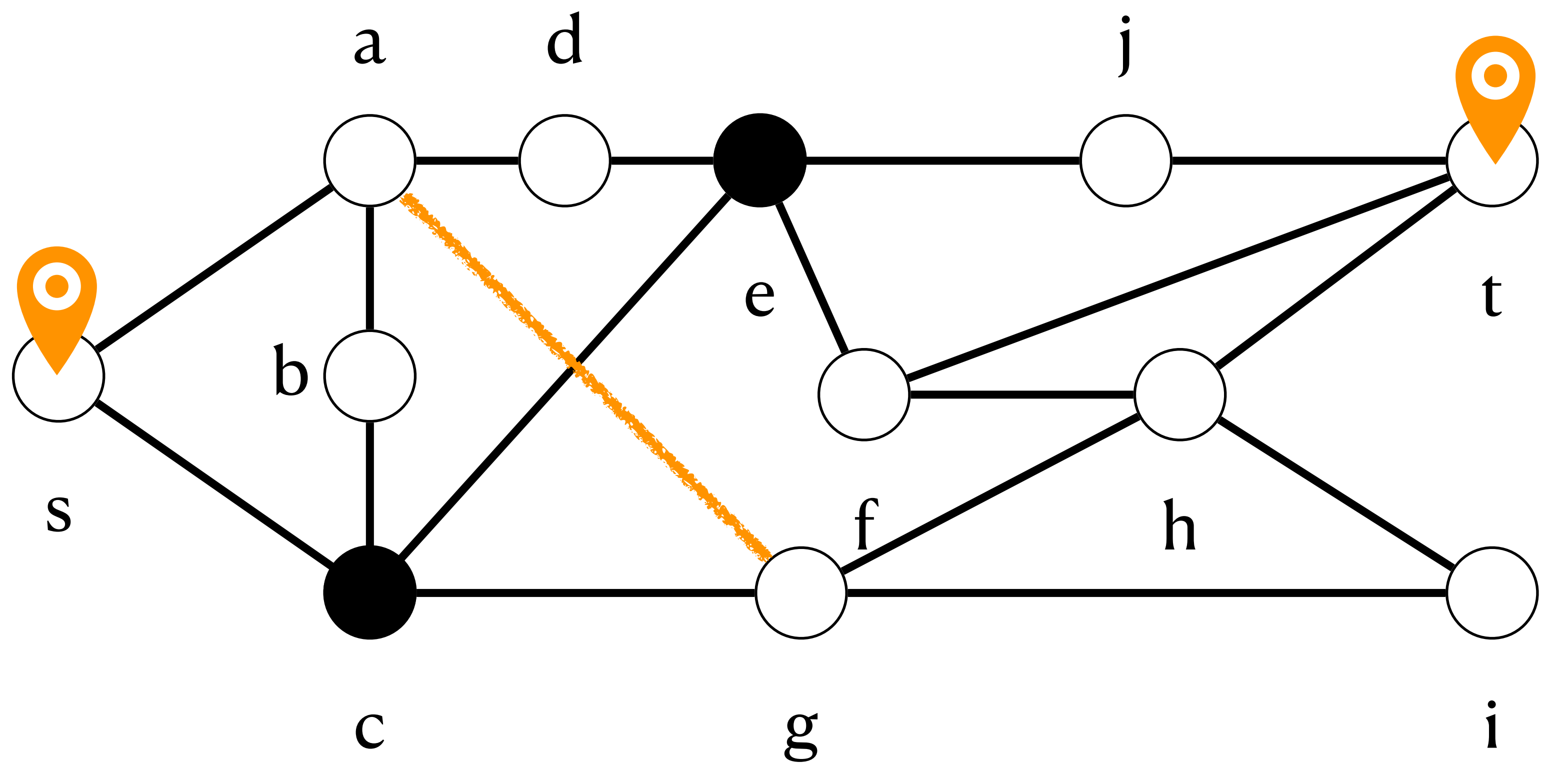

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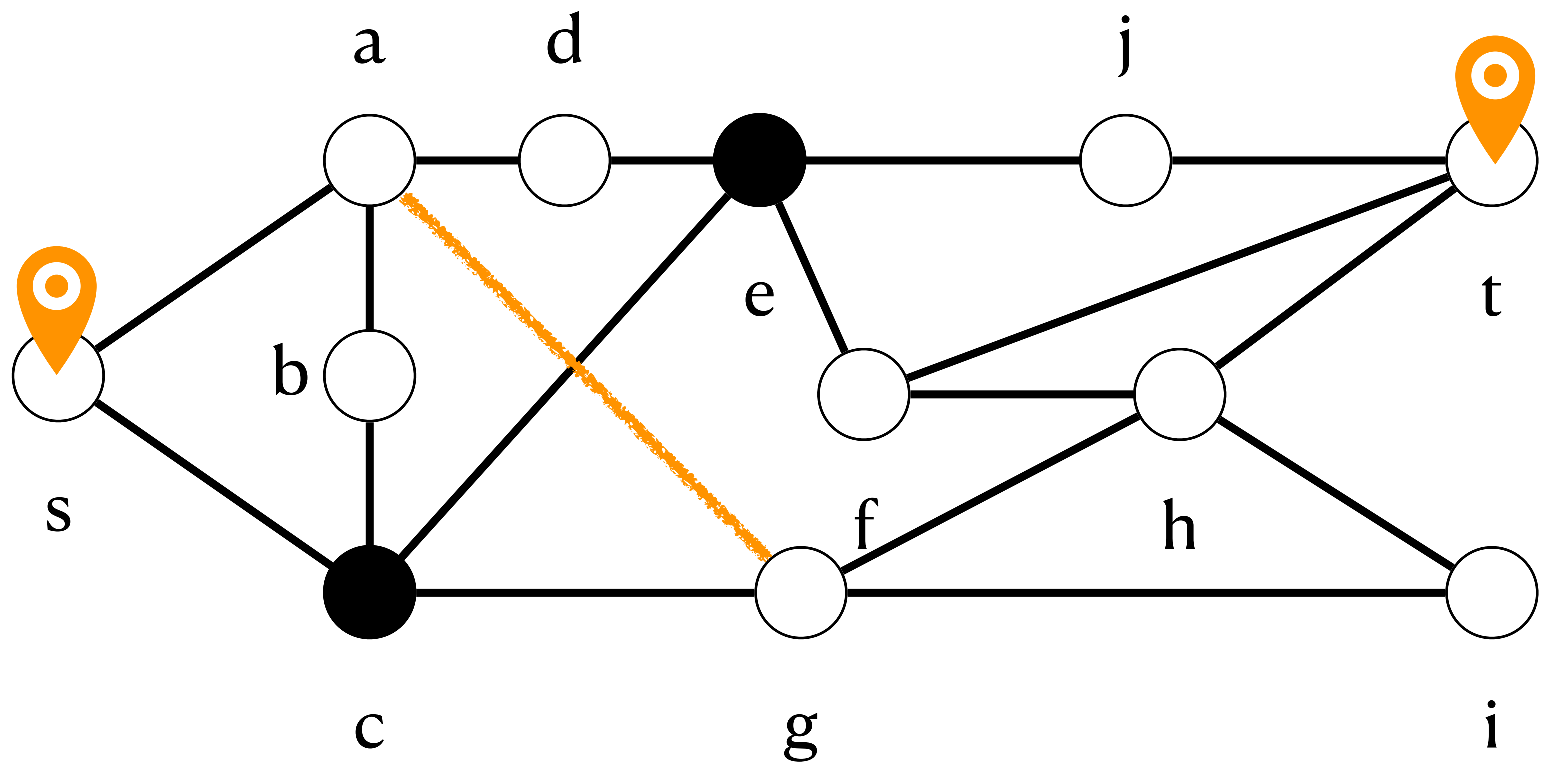

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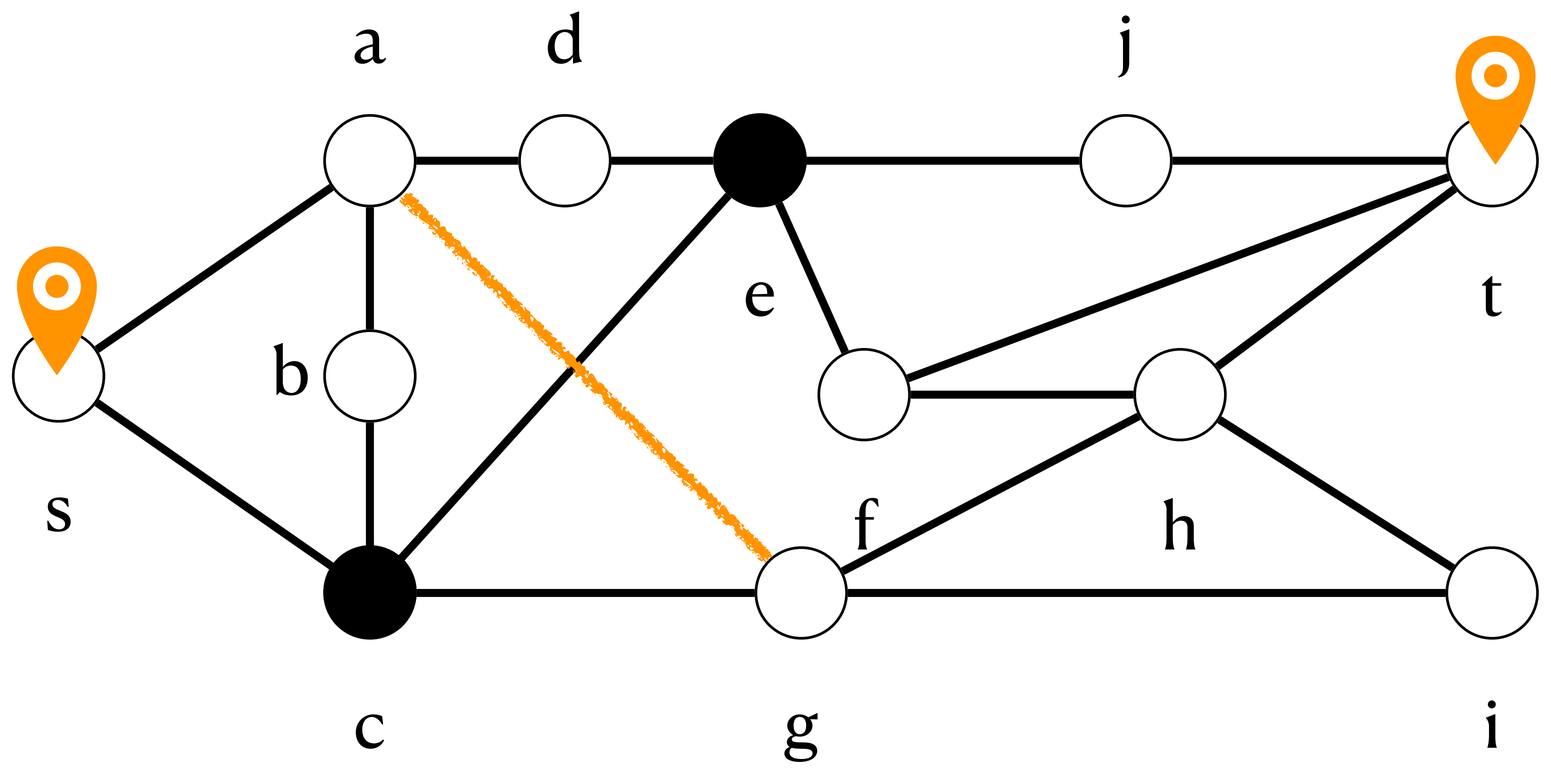

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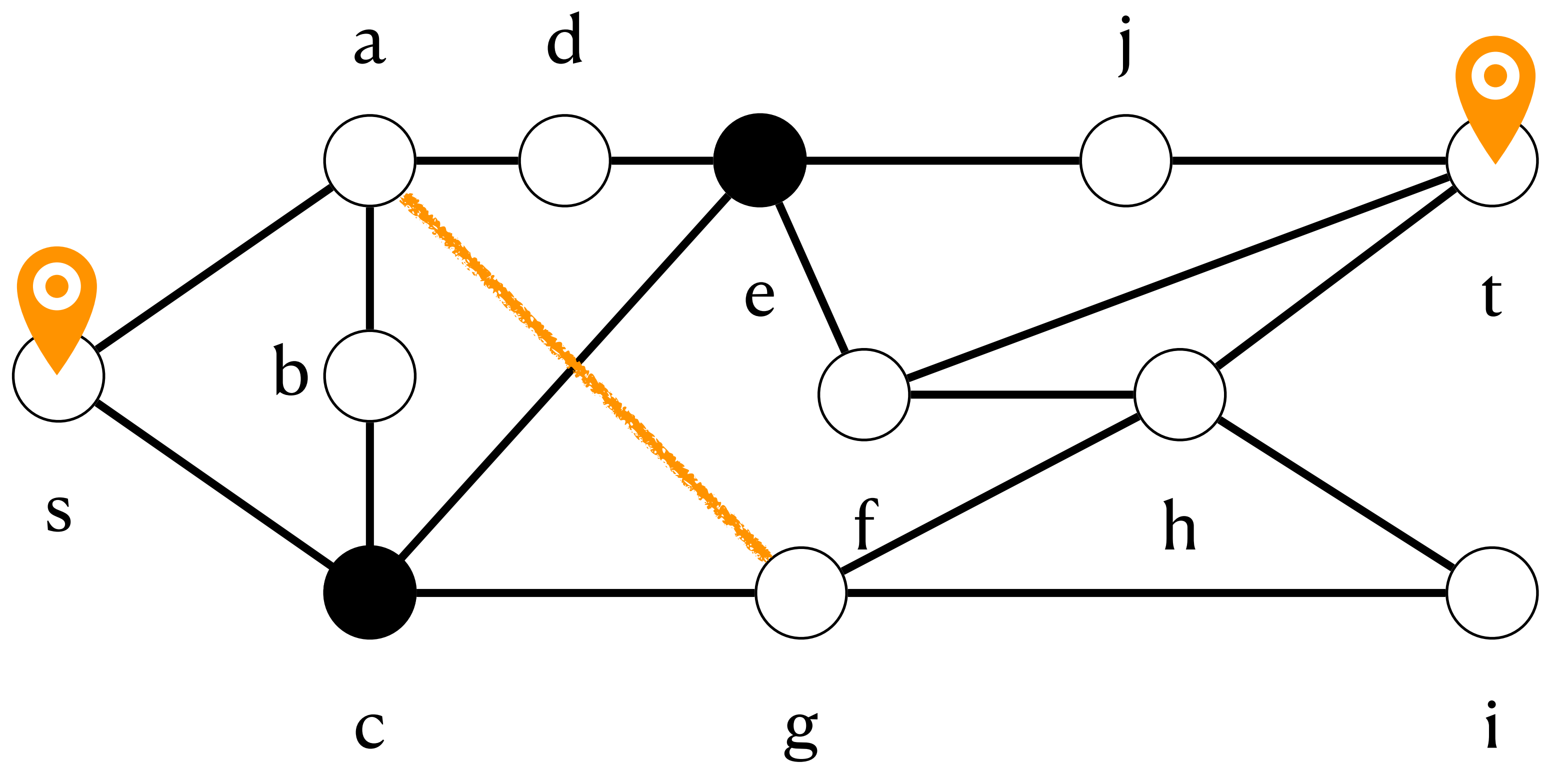

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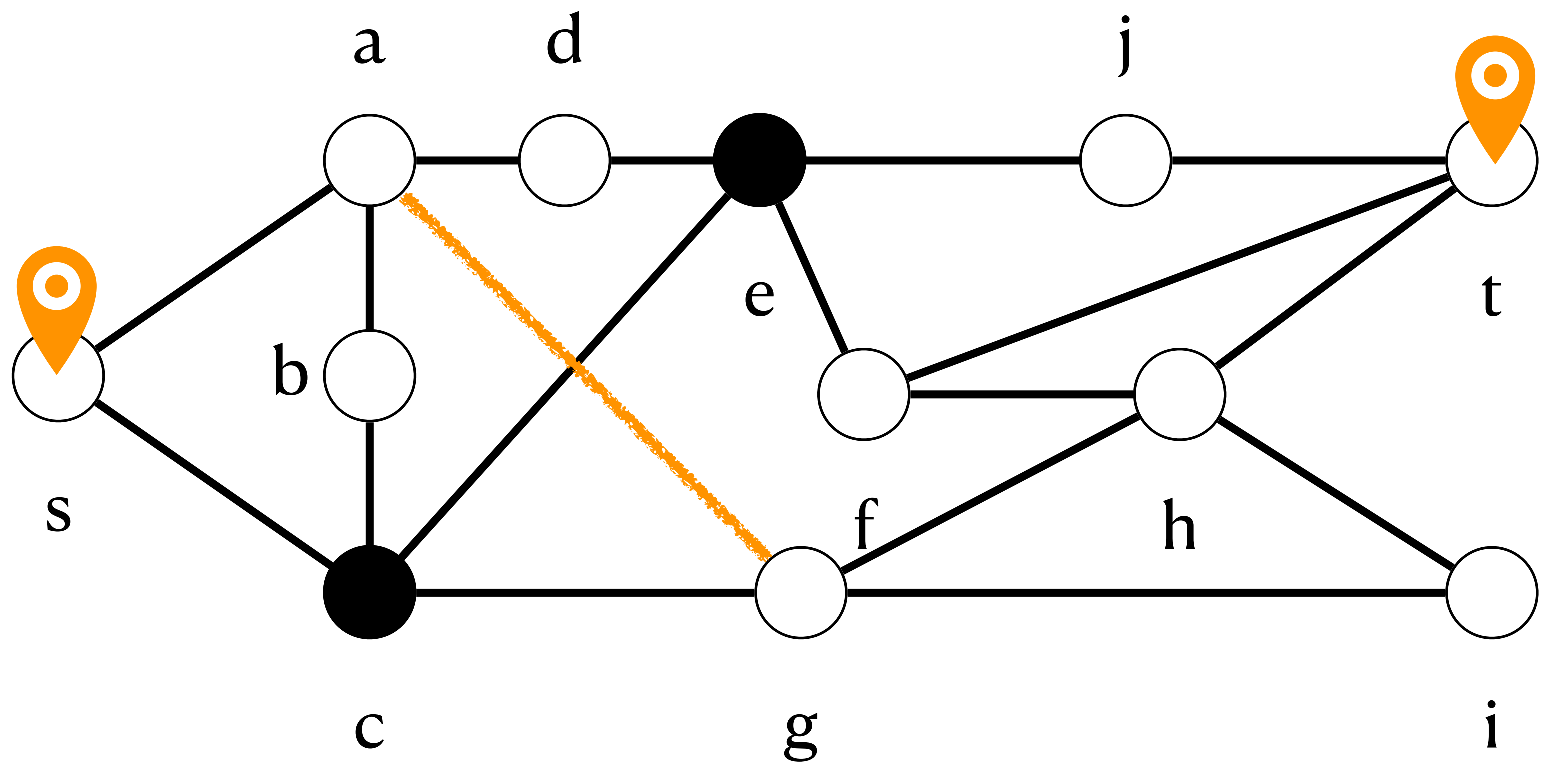

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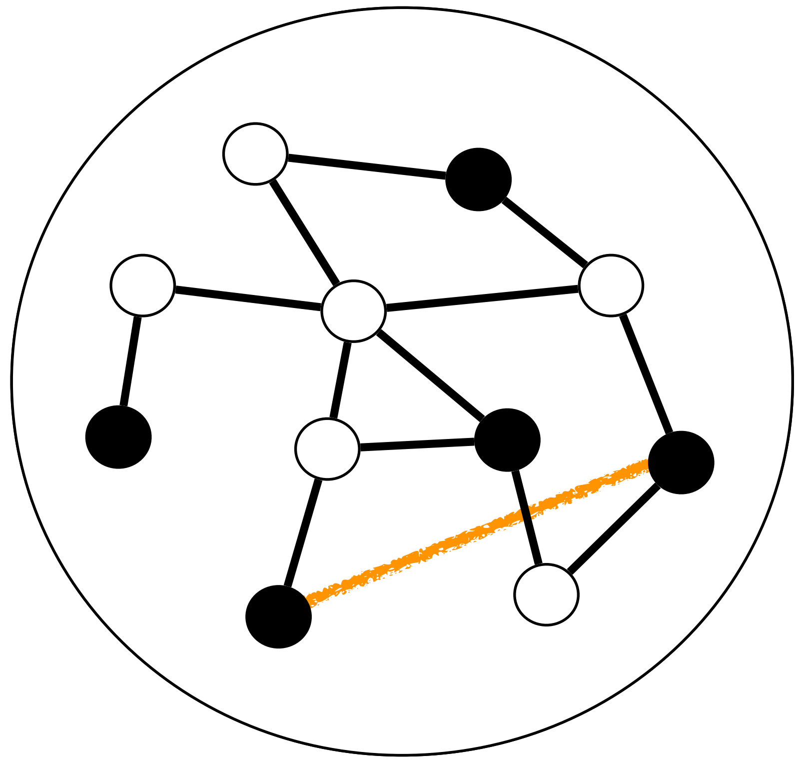

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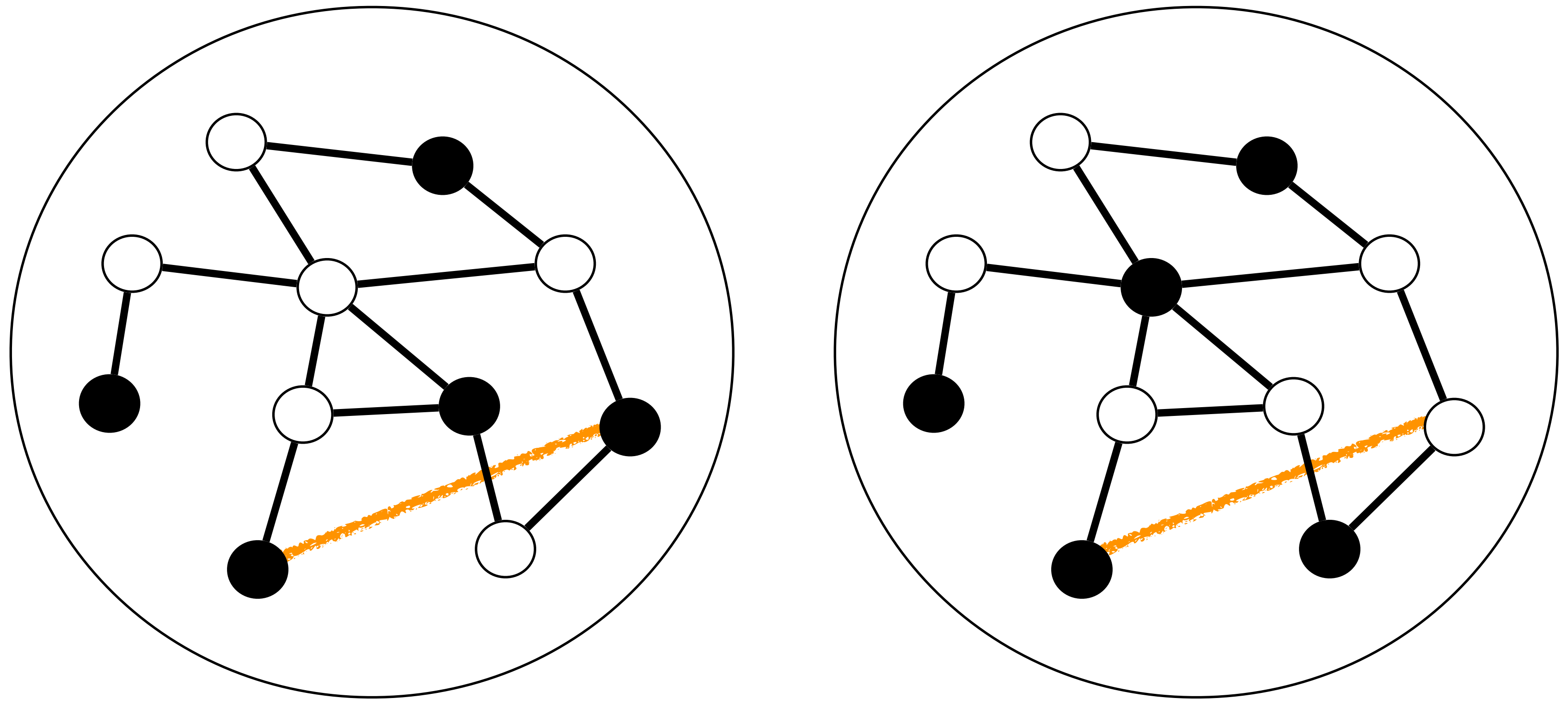

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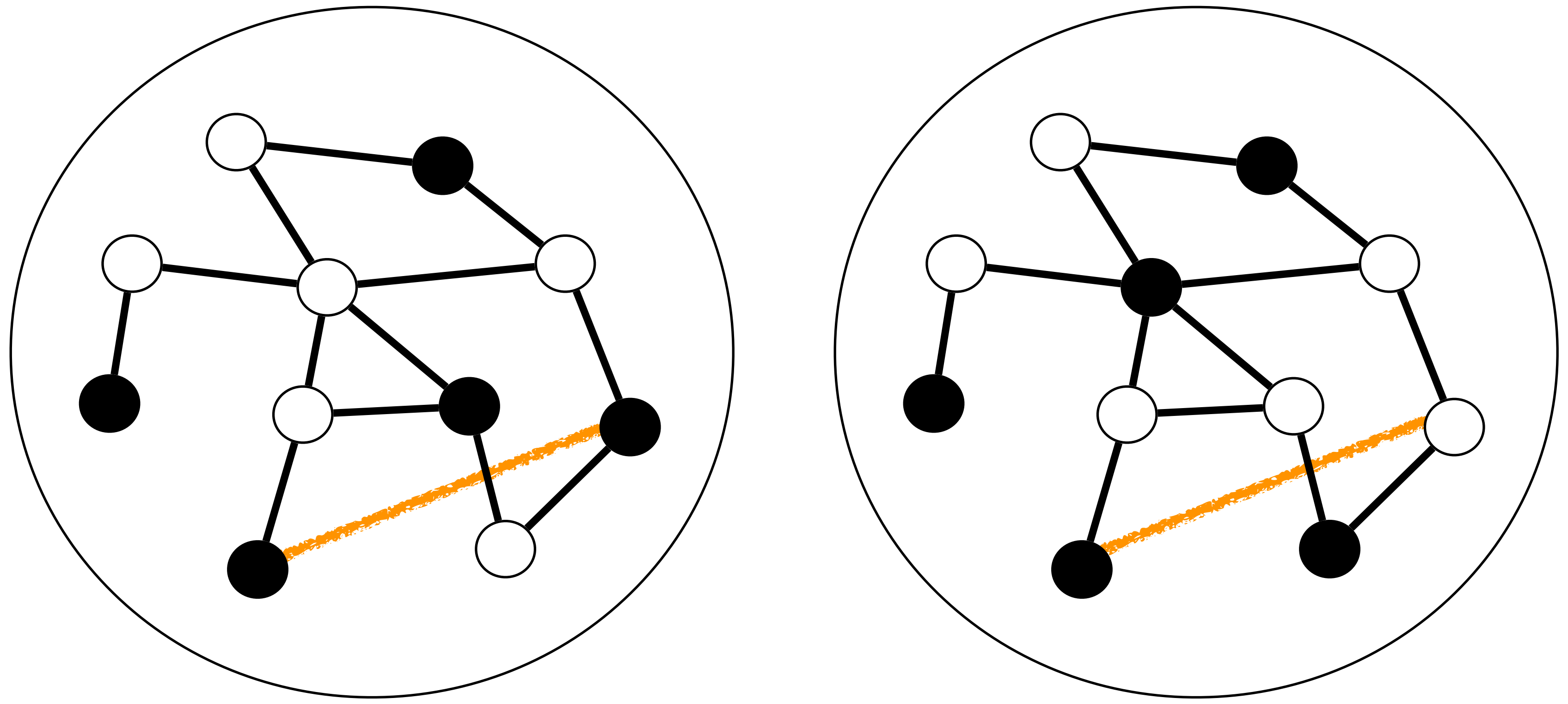

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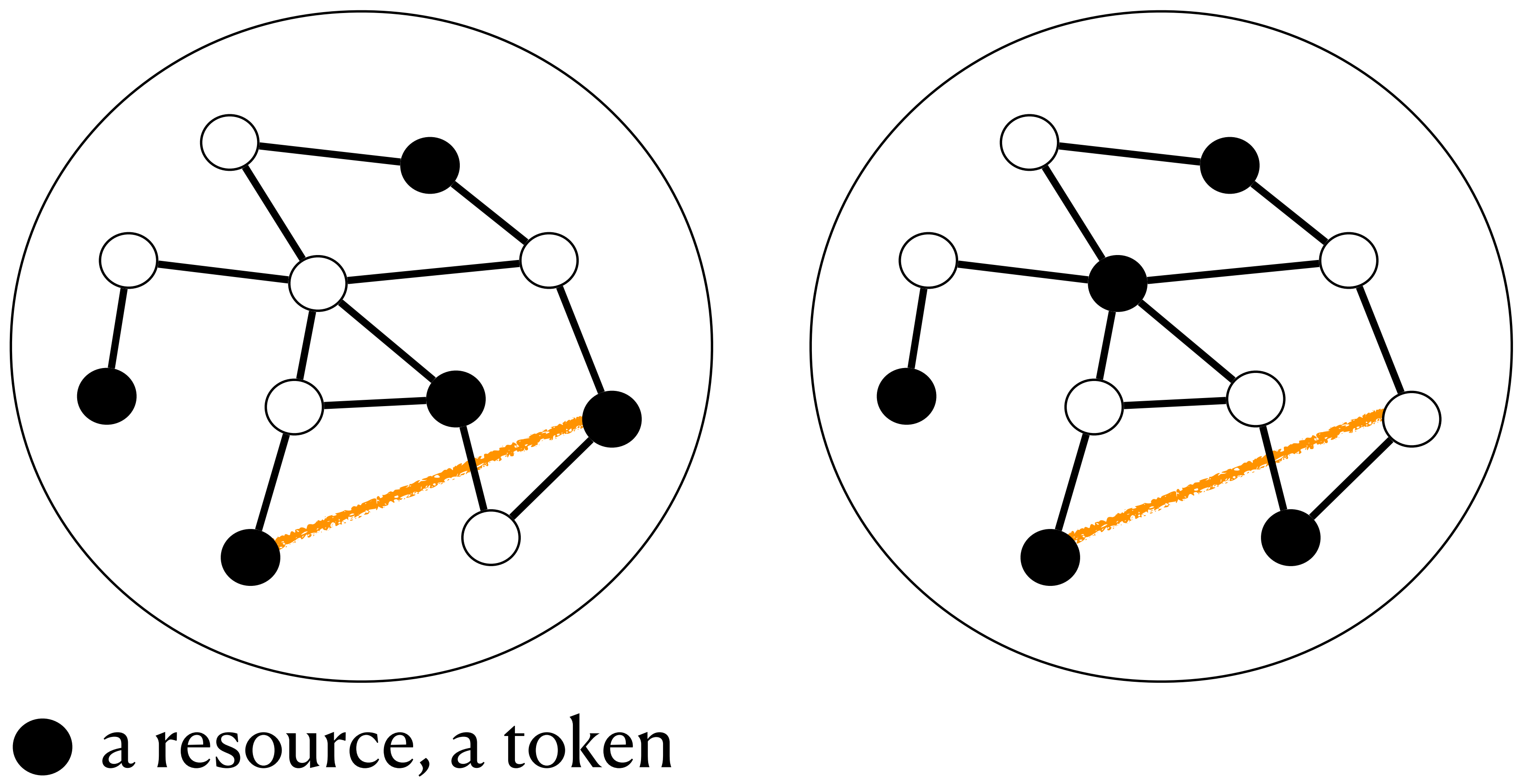

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Solution Discovery Algorithms **are also different** from..

I. Dynamic Graph Algorithms

II. Combinatorial Reconfiguration Algorithms

III. Local Search Algorithms

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- [Fellows et al, 2023], [Grobler et al. 2024] show **NP-hardness** for all these solution discovery problems.

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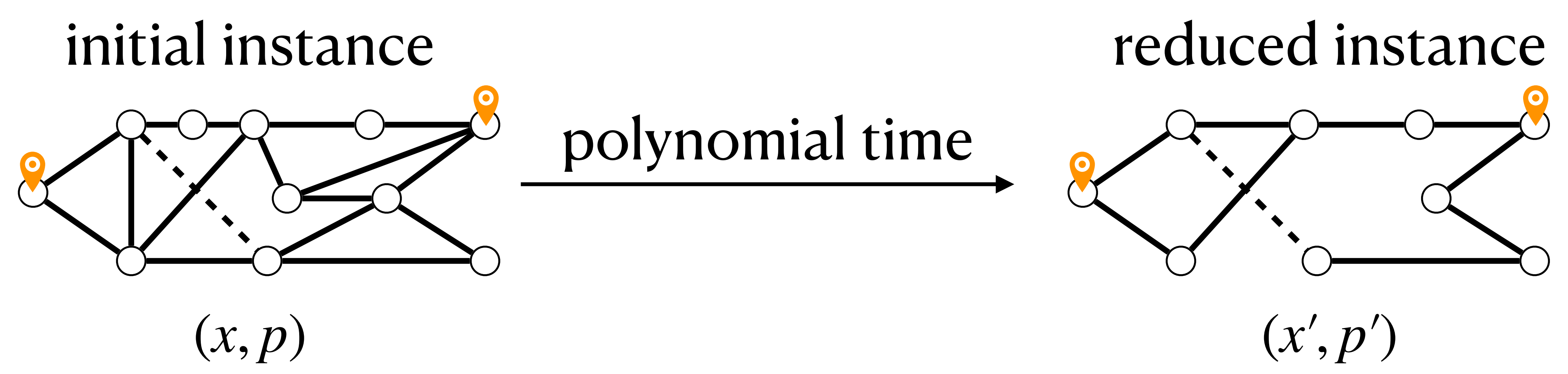
Algorithm A



p: parameter

x: instance for classical problem

Parameterized and Kernelization Complexities

- A *parameterized problem* with an algorithm that runs in $g(p) \cdot |x|^c$ time, where g is a function, is in FPT.
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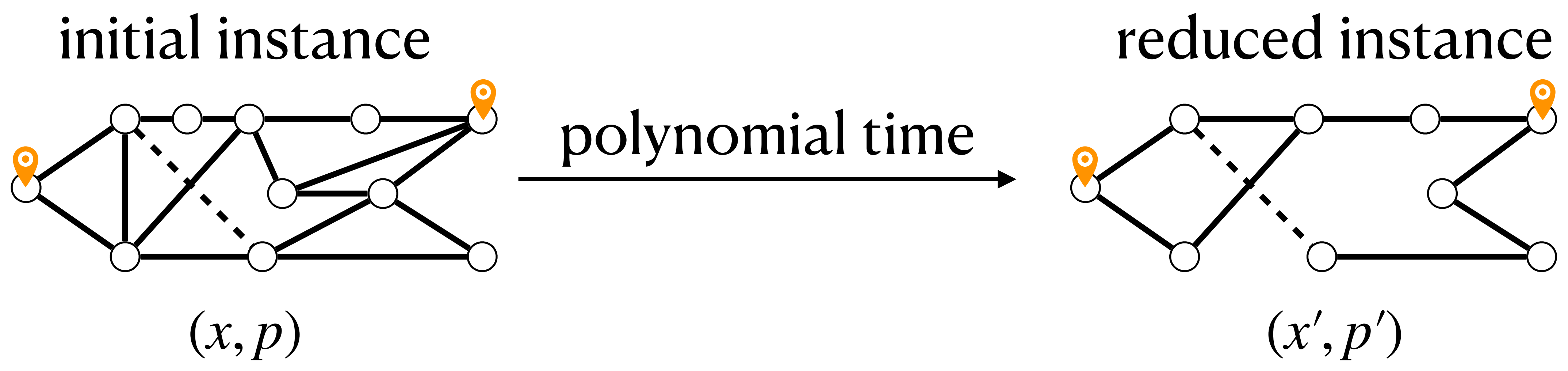

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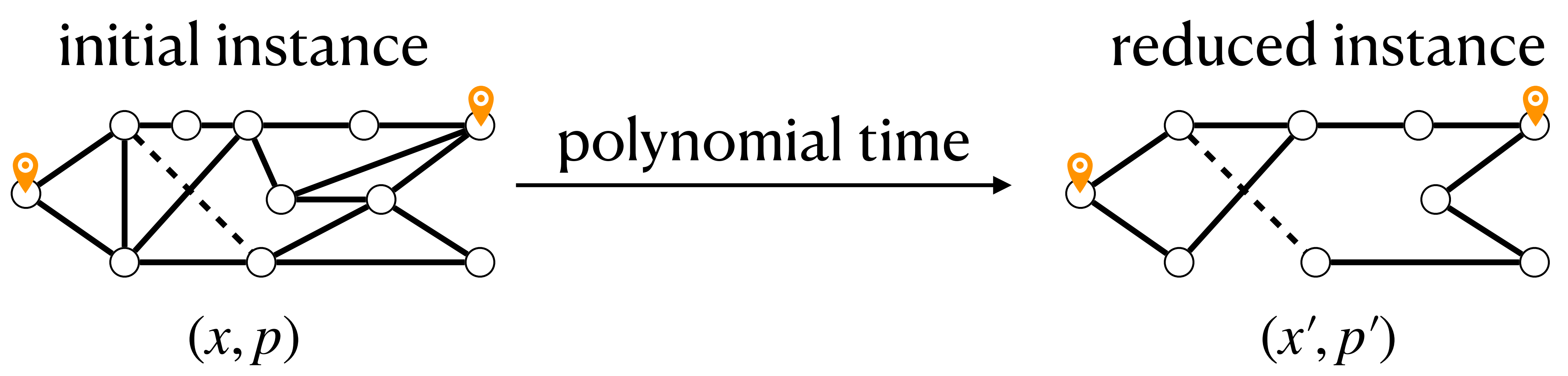

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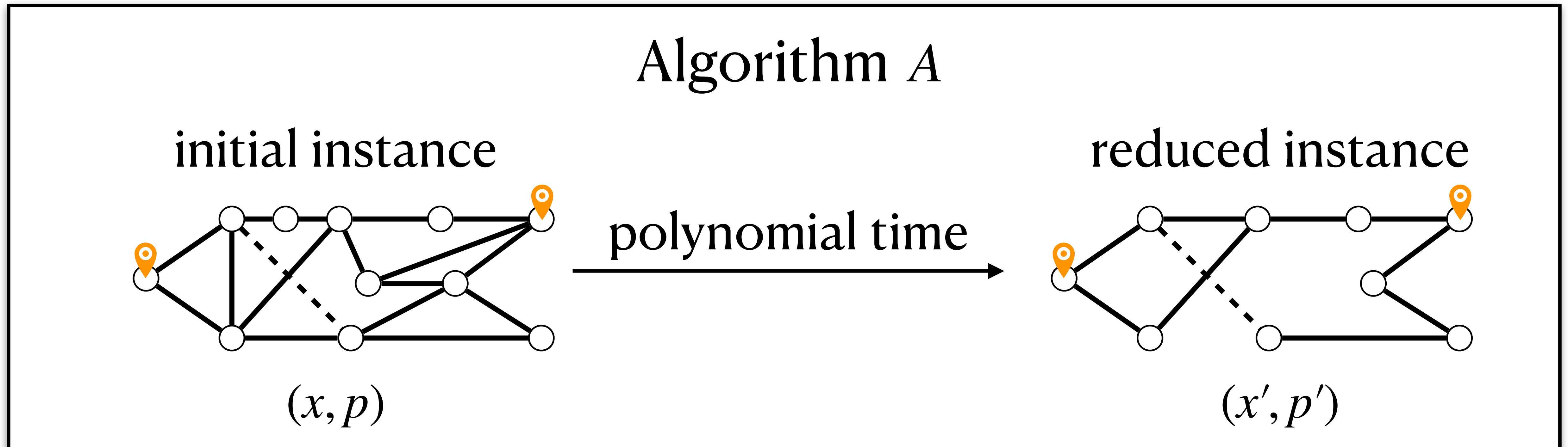

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Our Results

- We studied the kernelization of the considered problems, unlike [Fellows et al, 2023] and [Grobler et al. 2024].

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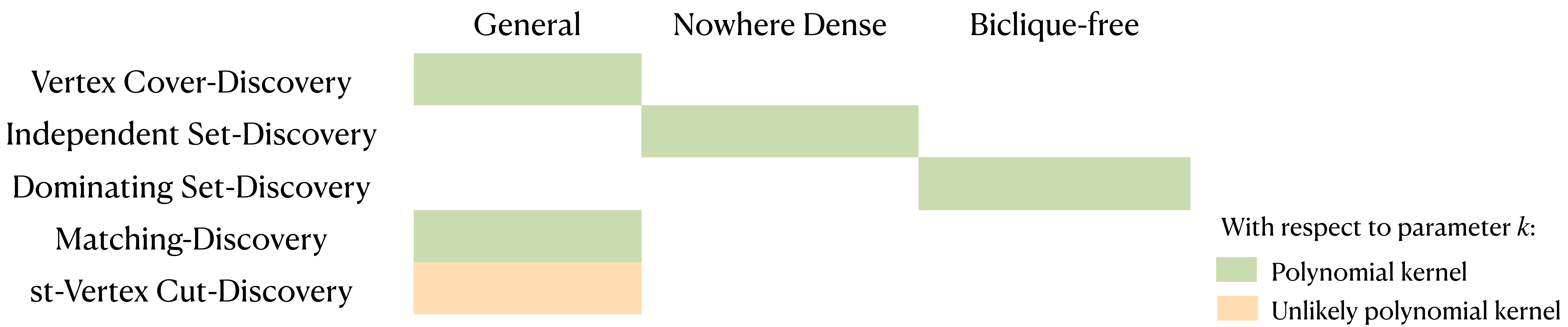

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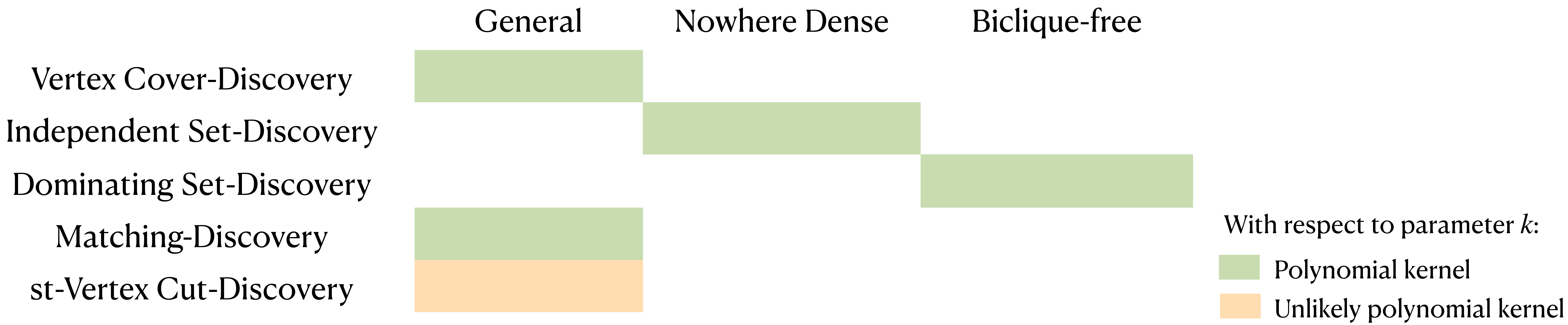

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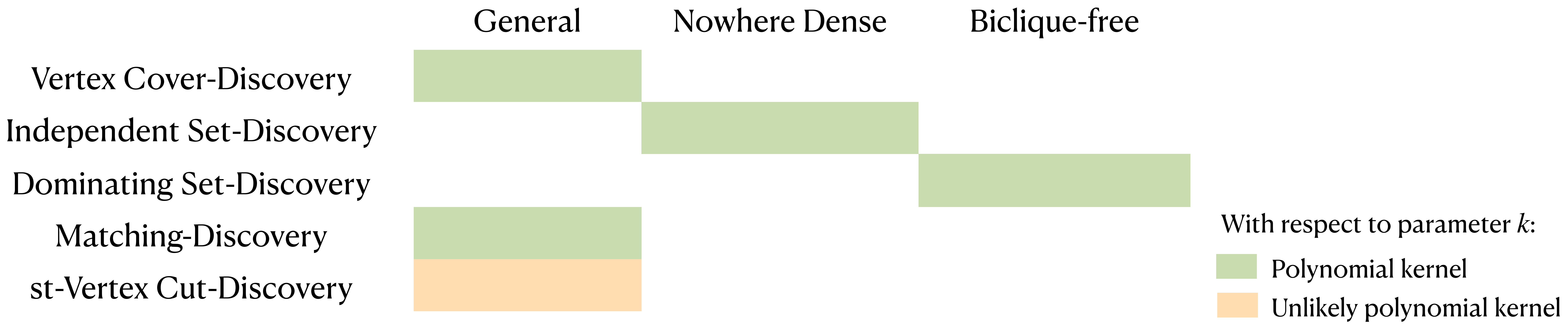

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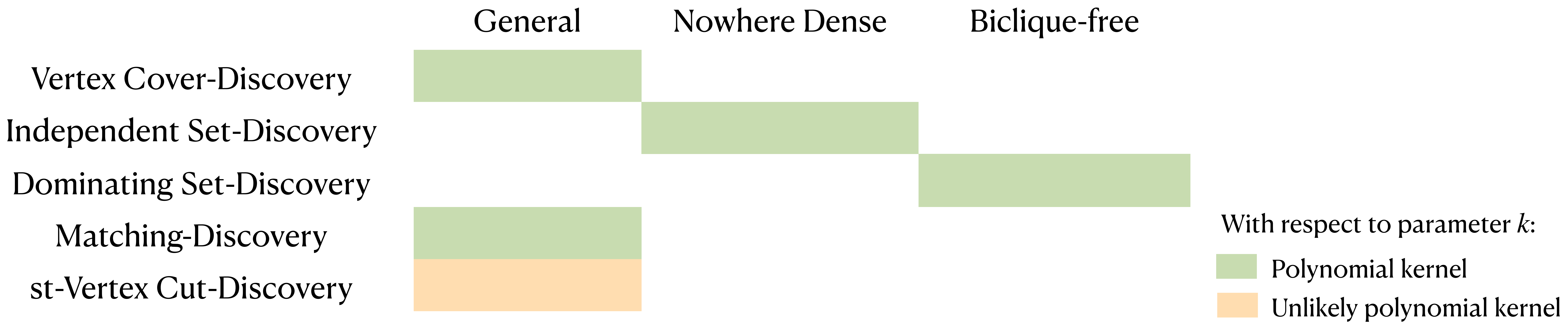

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st-Vertex Cut Discovery with Respect to Parameter k

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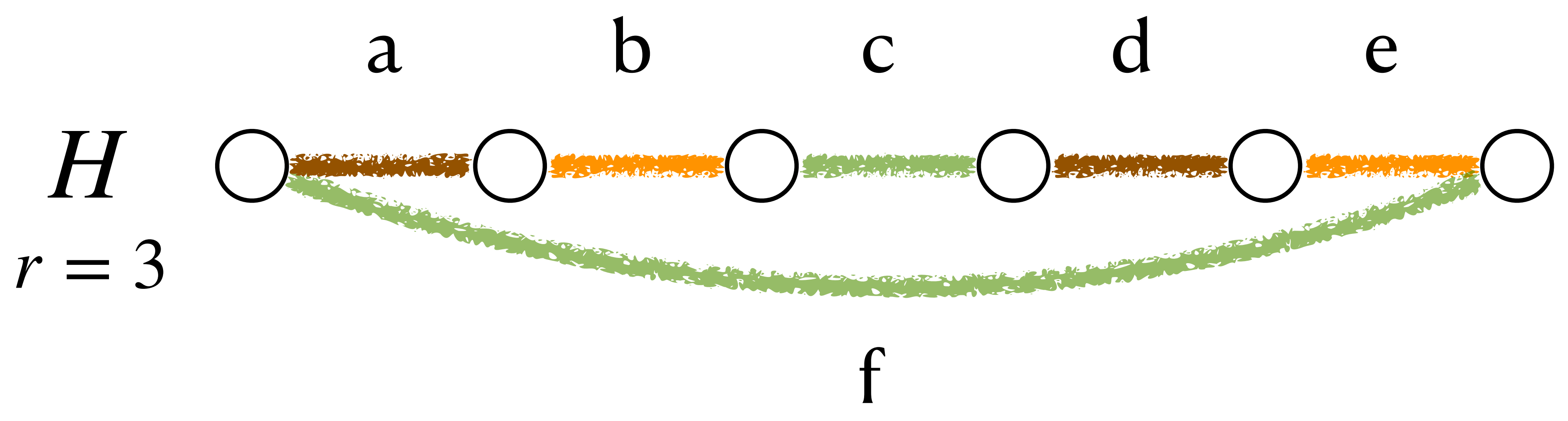

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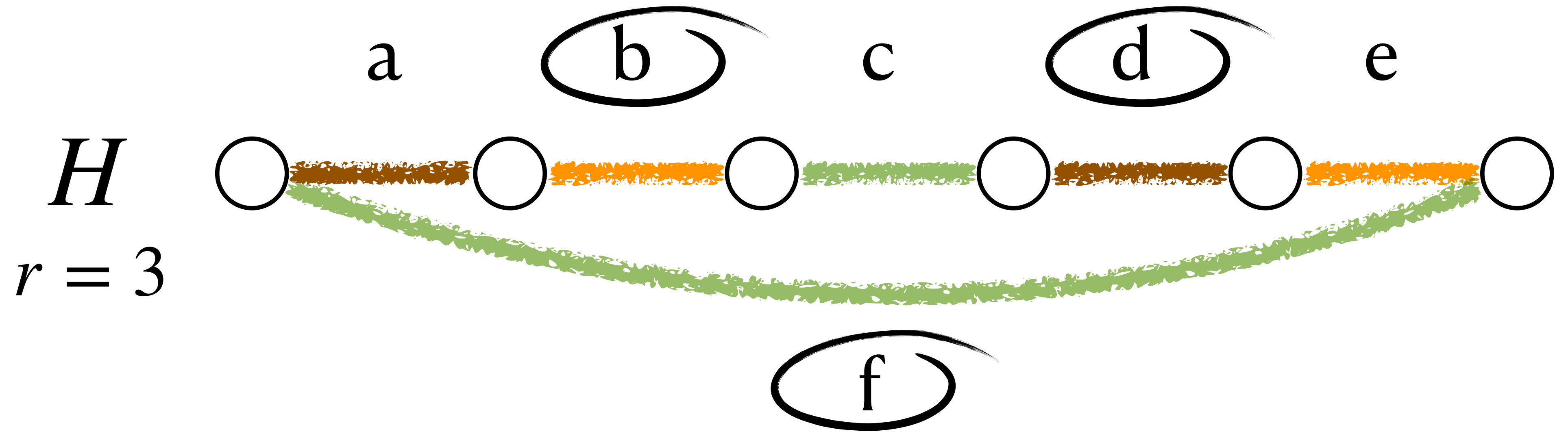

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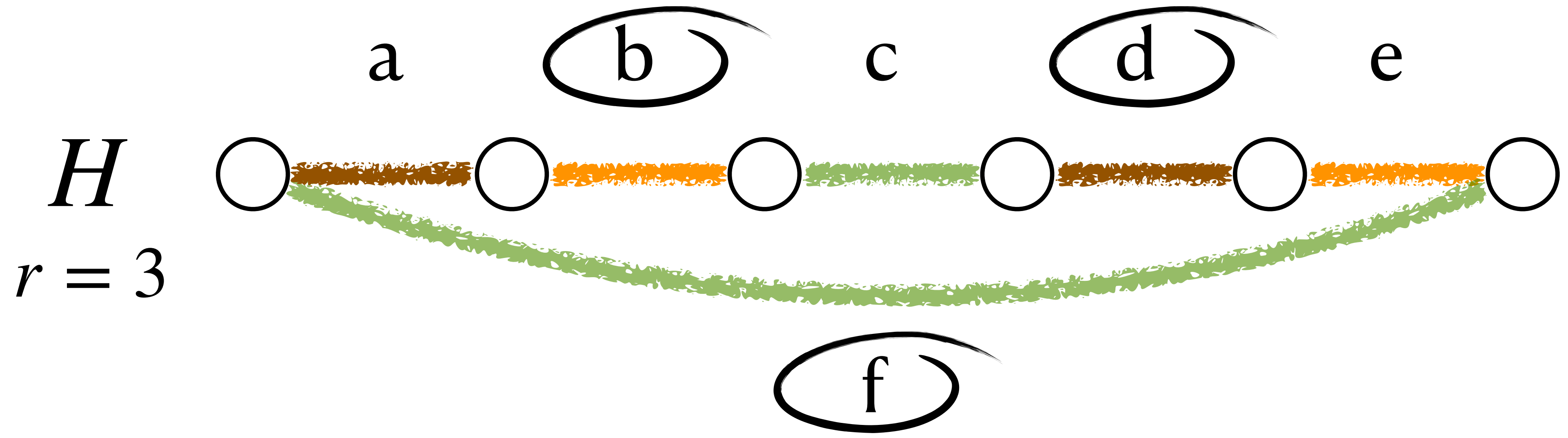

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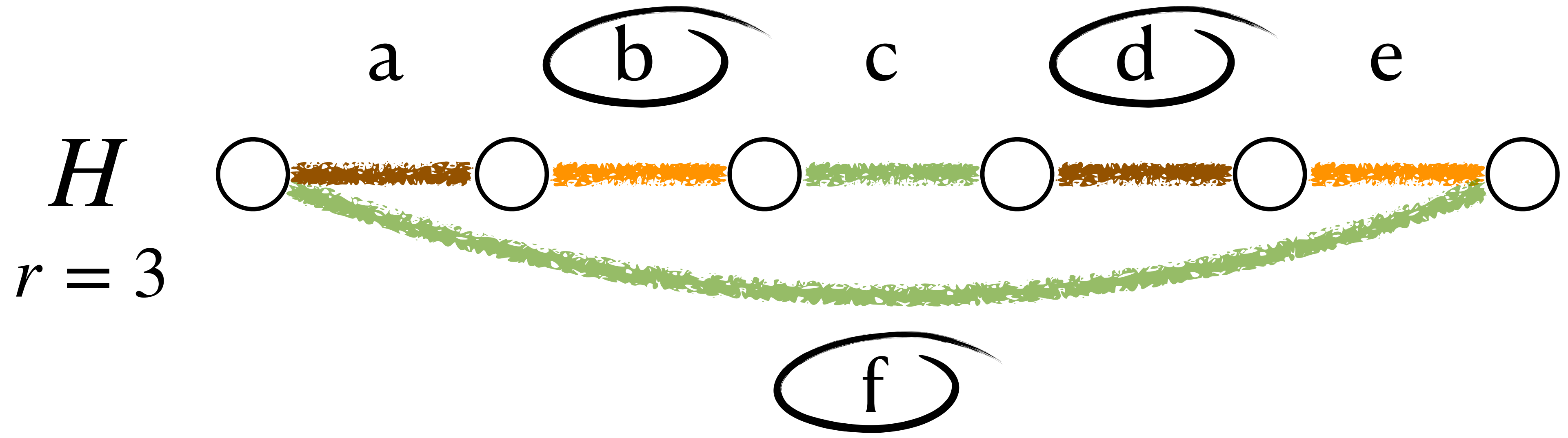

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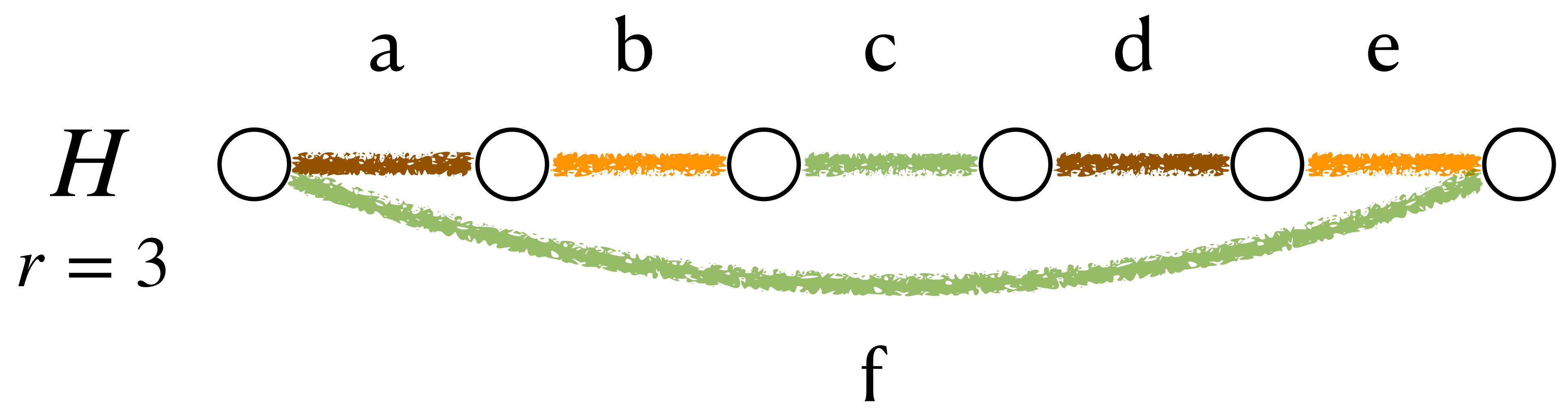
Reduction from Rainbow Matching to st-Vertex Cut Discovery



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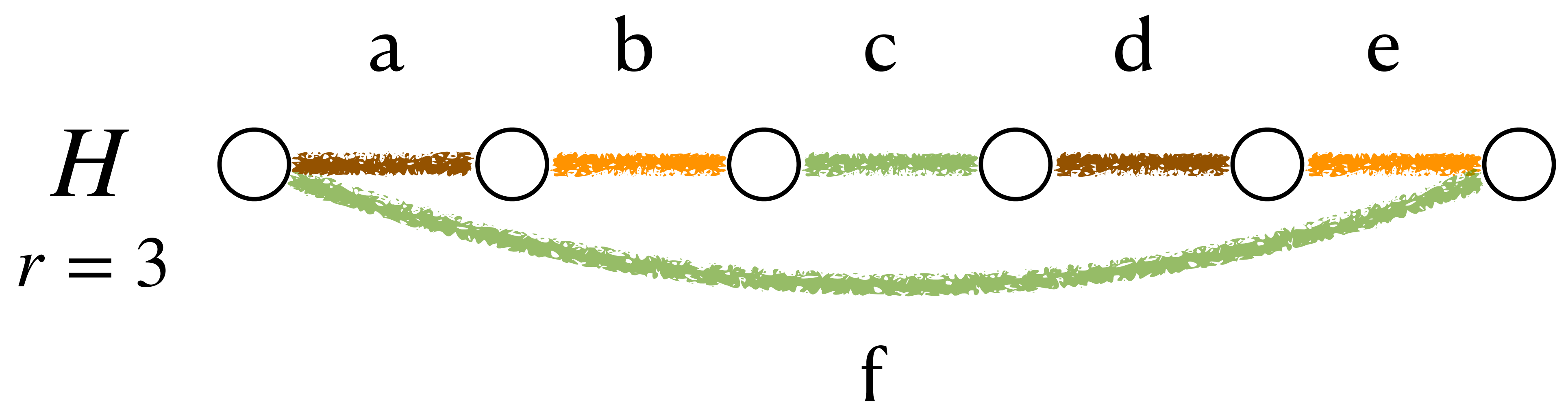

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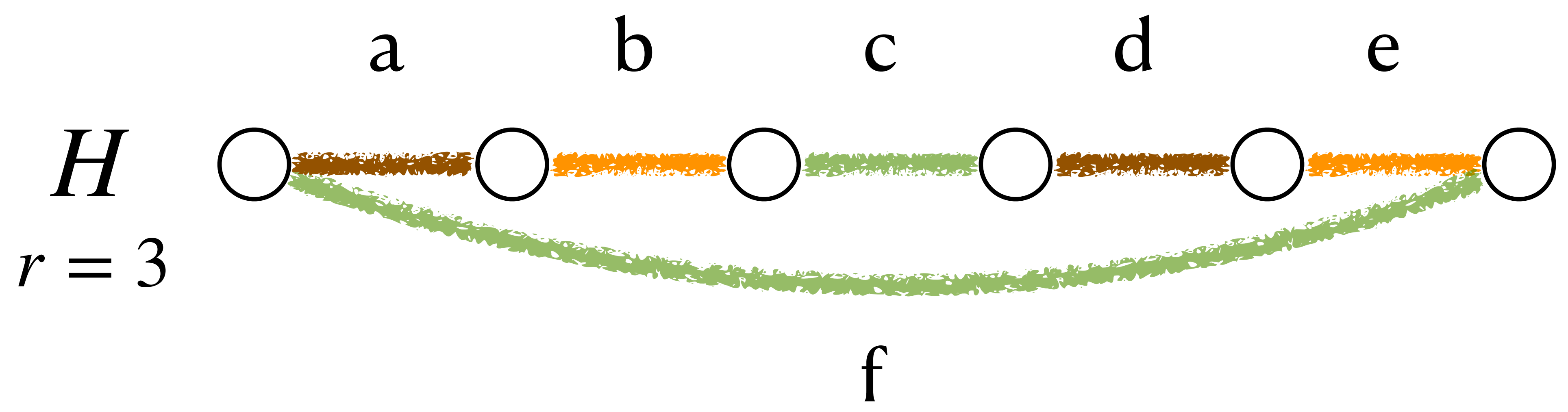

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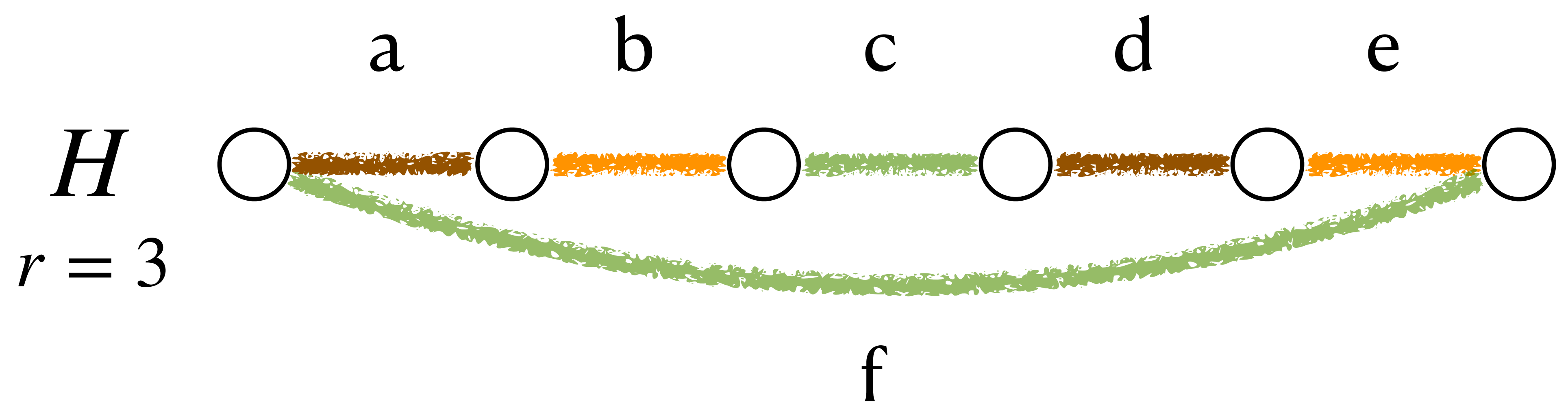

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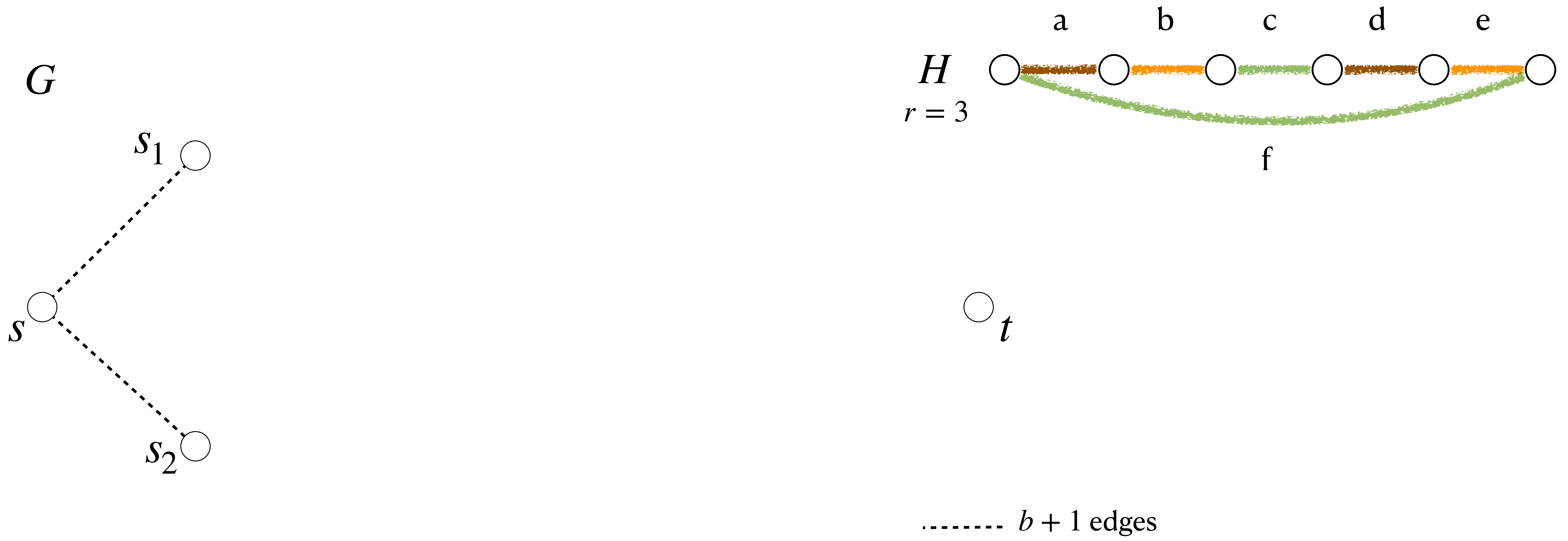

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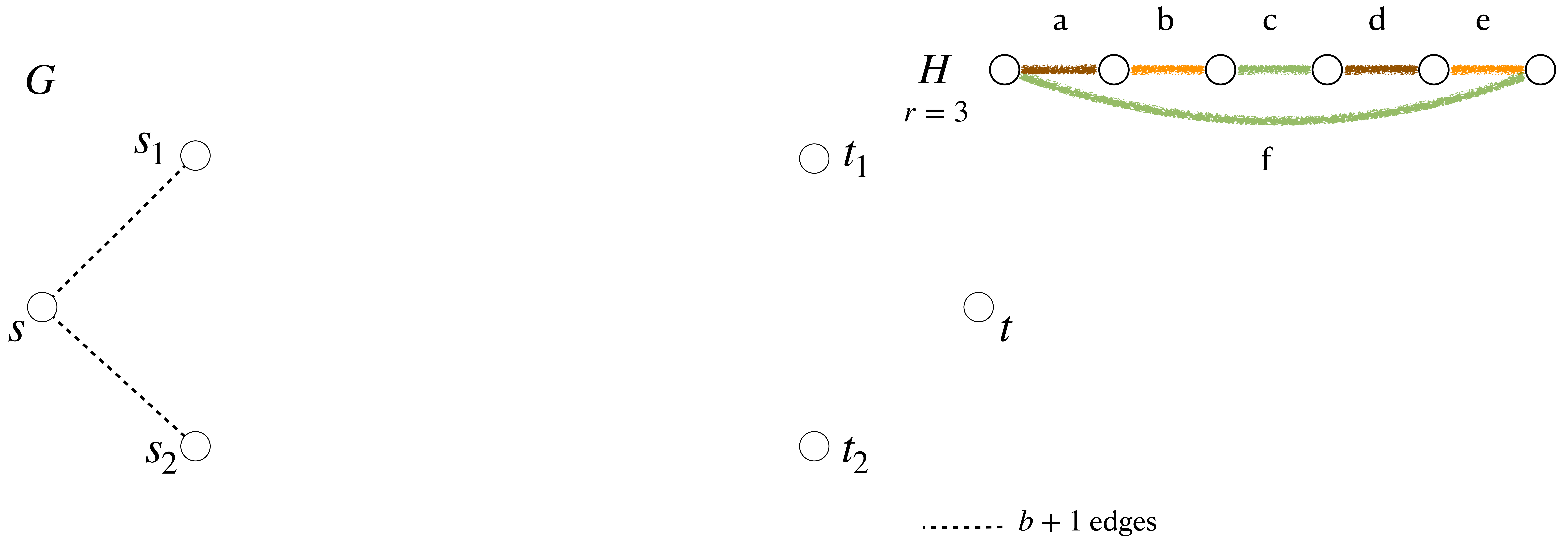

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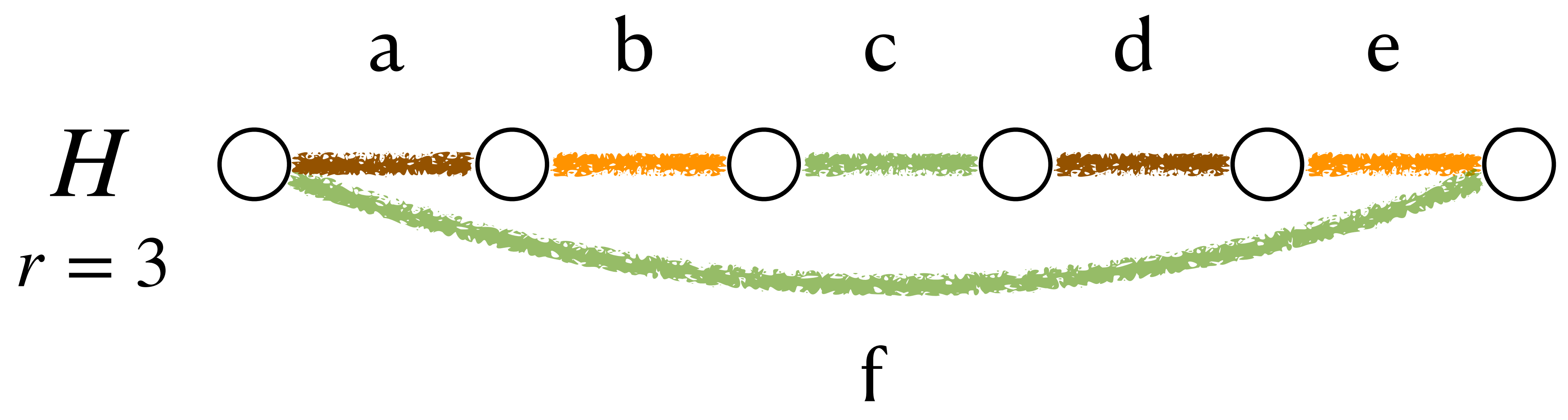

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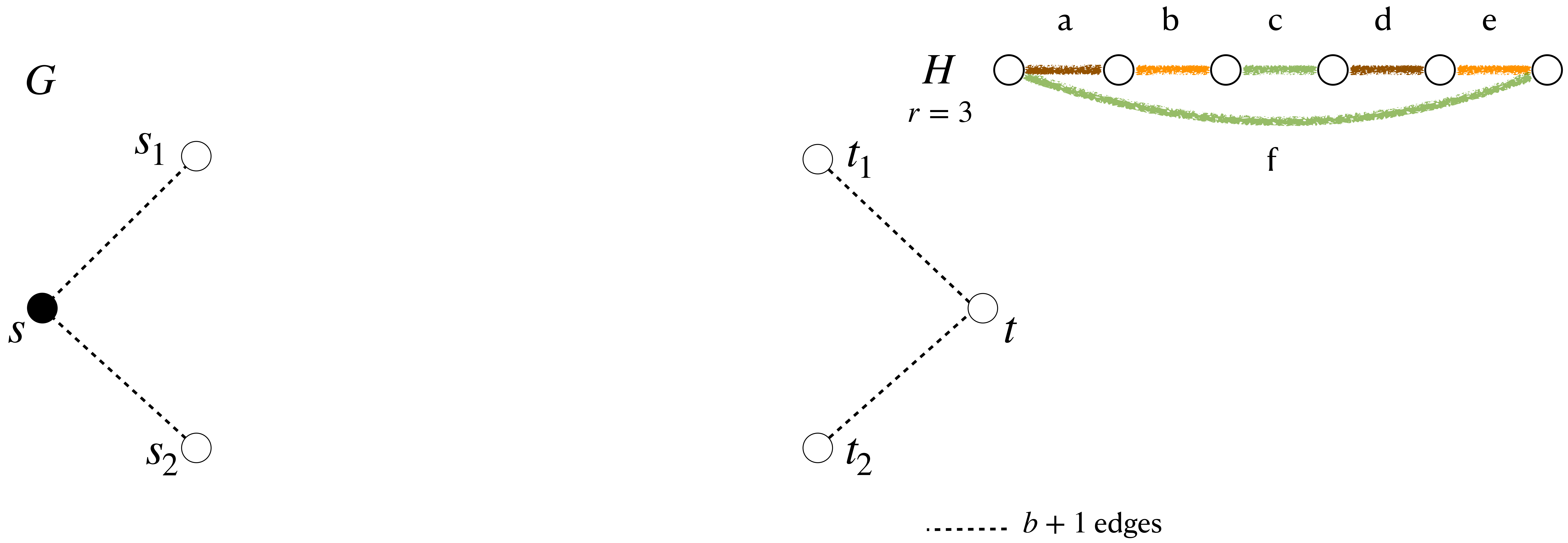

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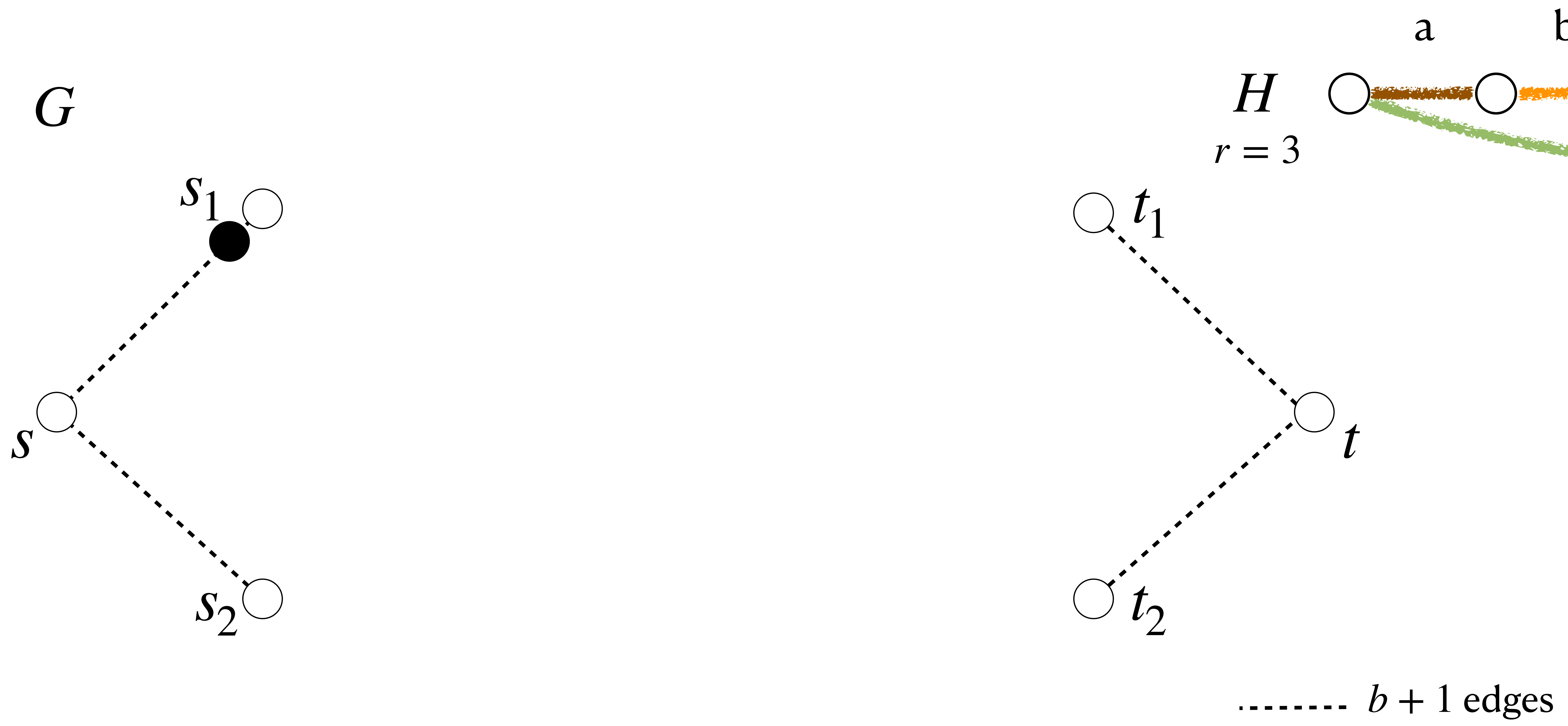

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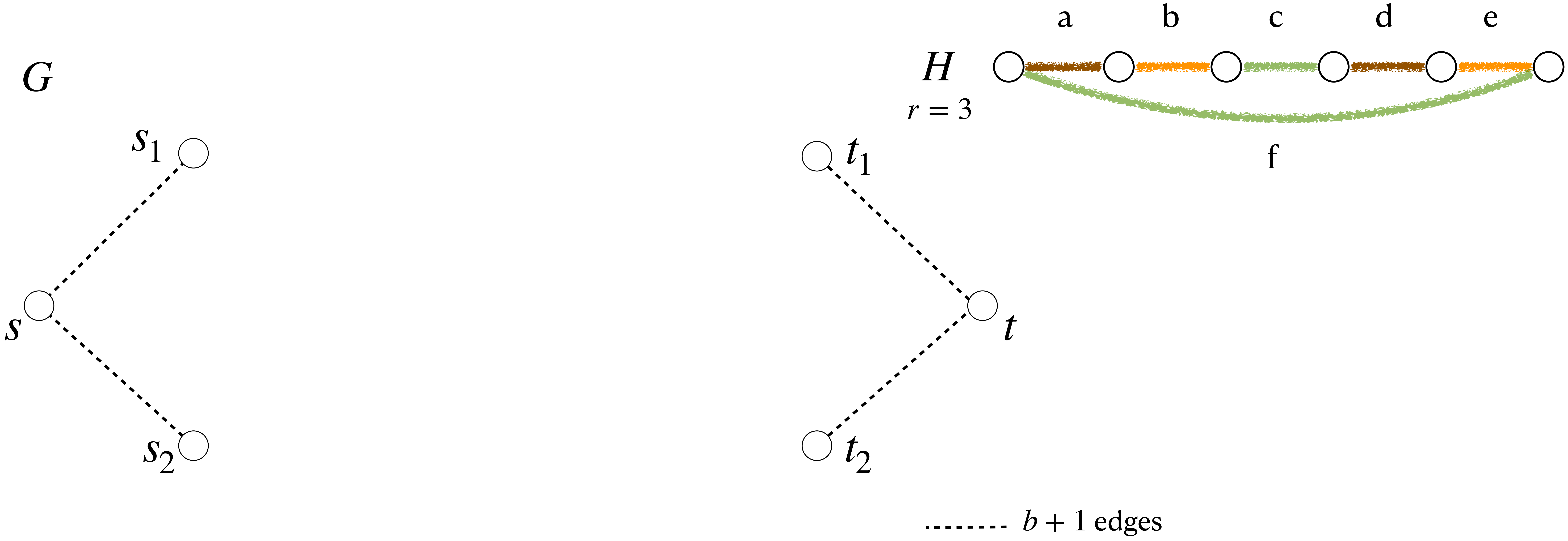

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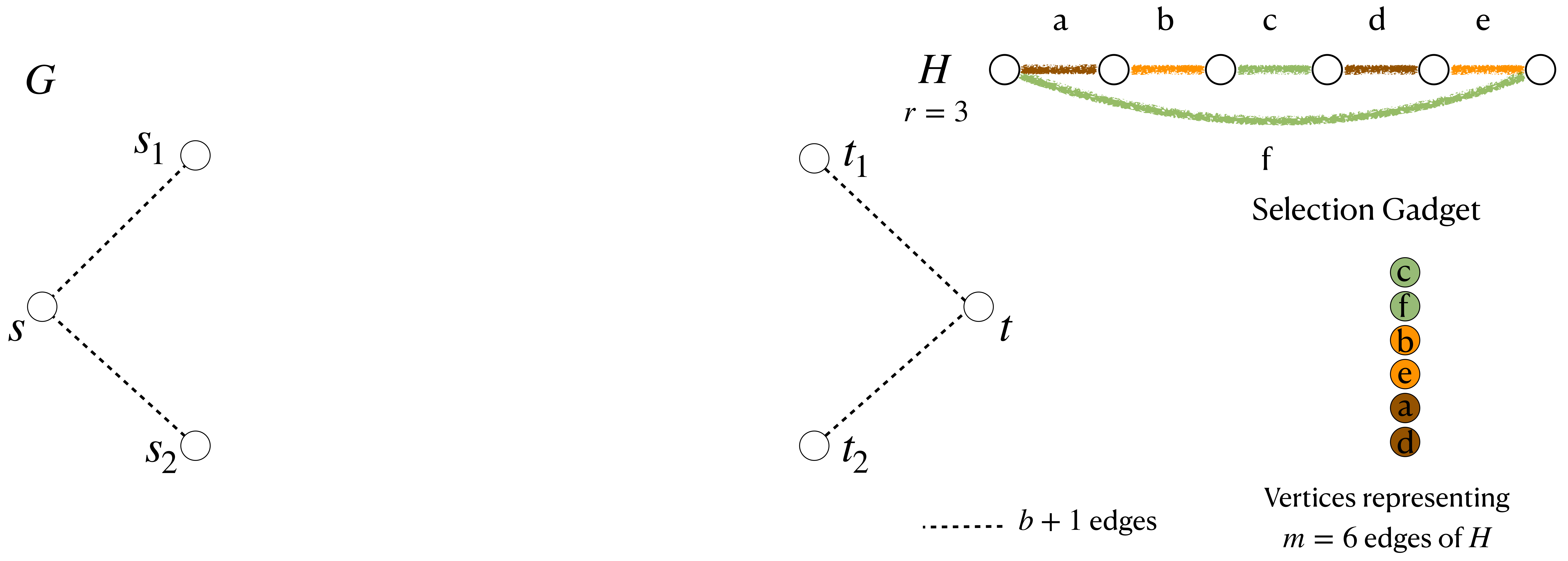

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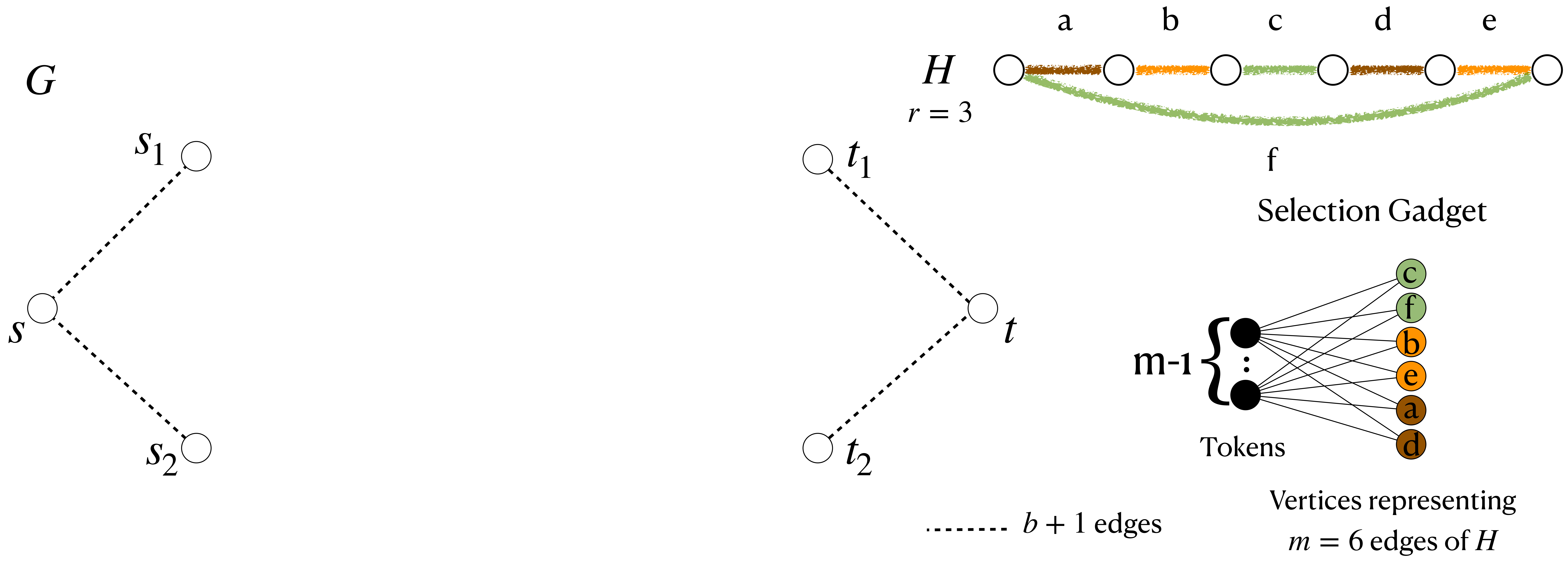

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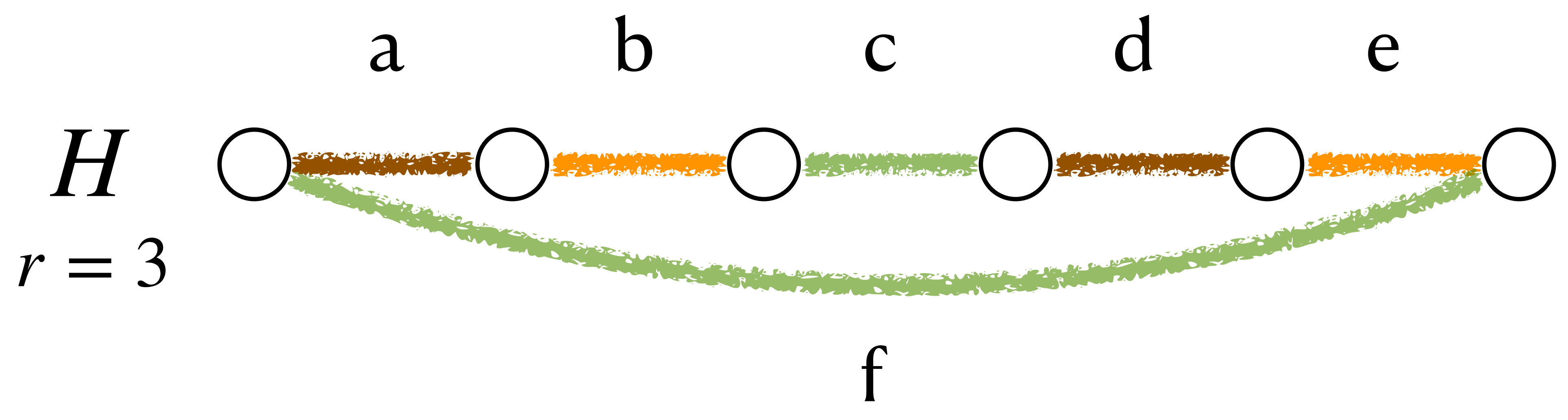
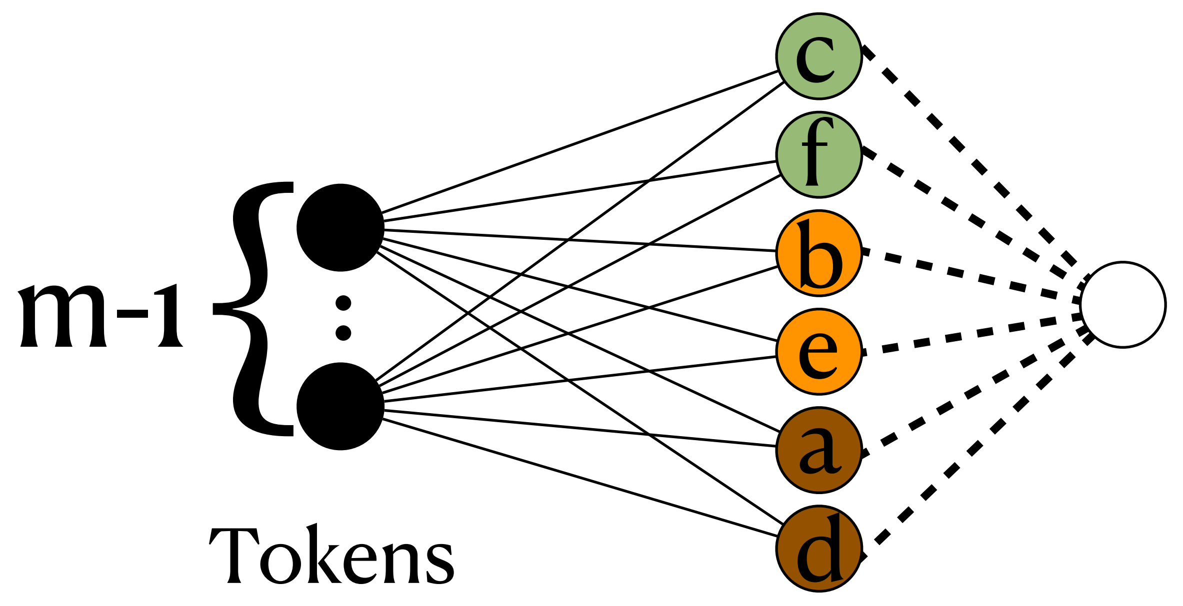


Selection Gadget



$b + 1$ edges

Vertices representing $m = 6$ edges of H

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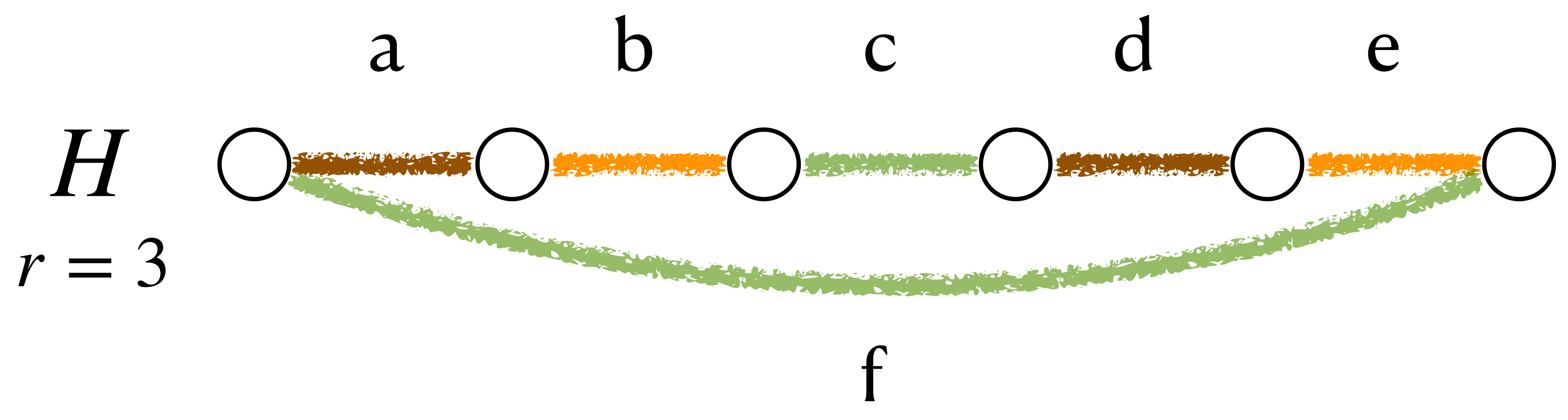

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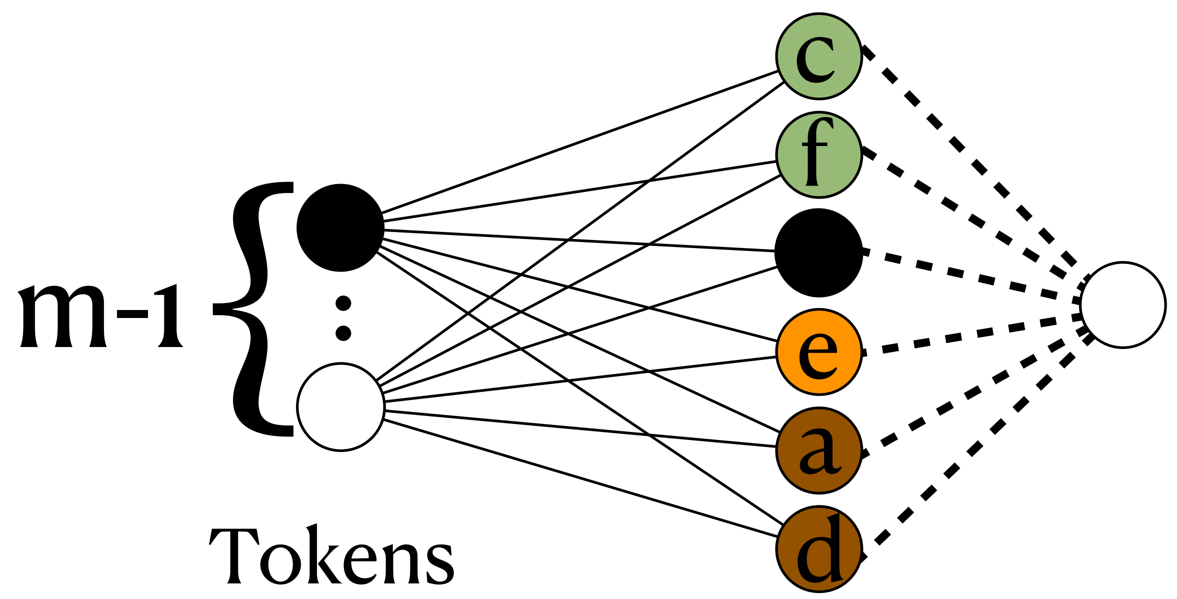

Tokens

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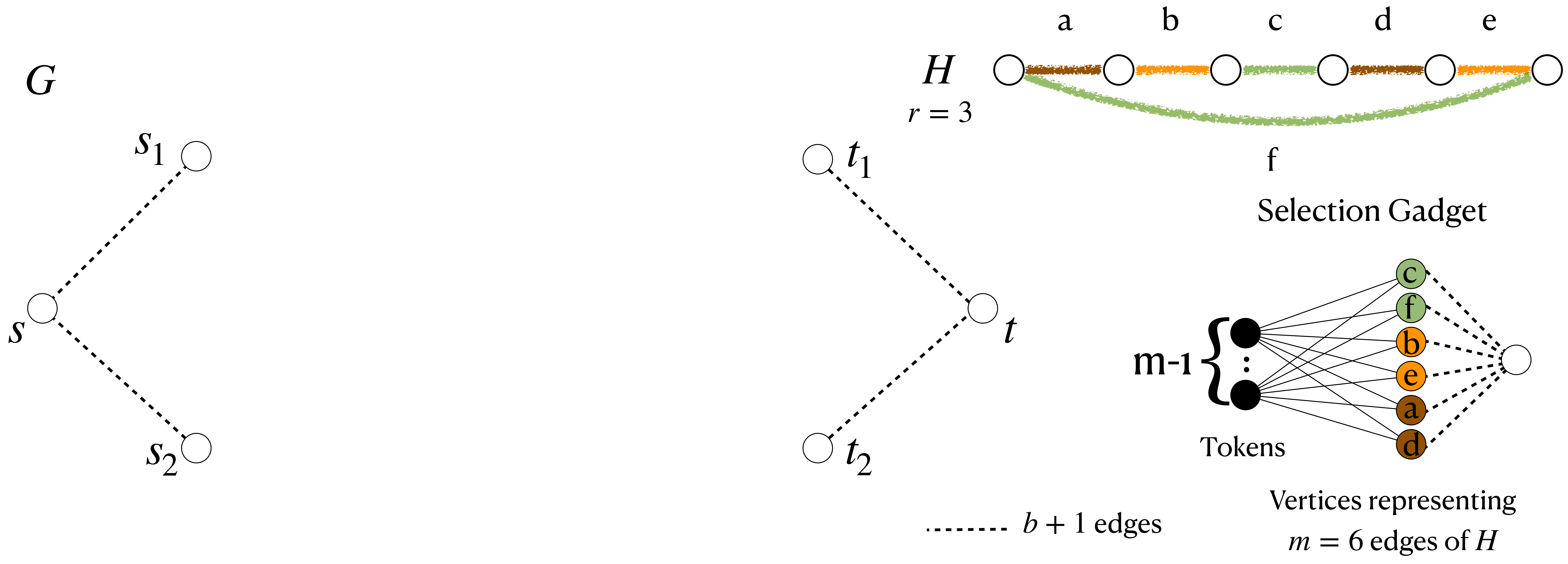

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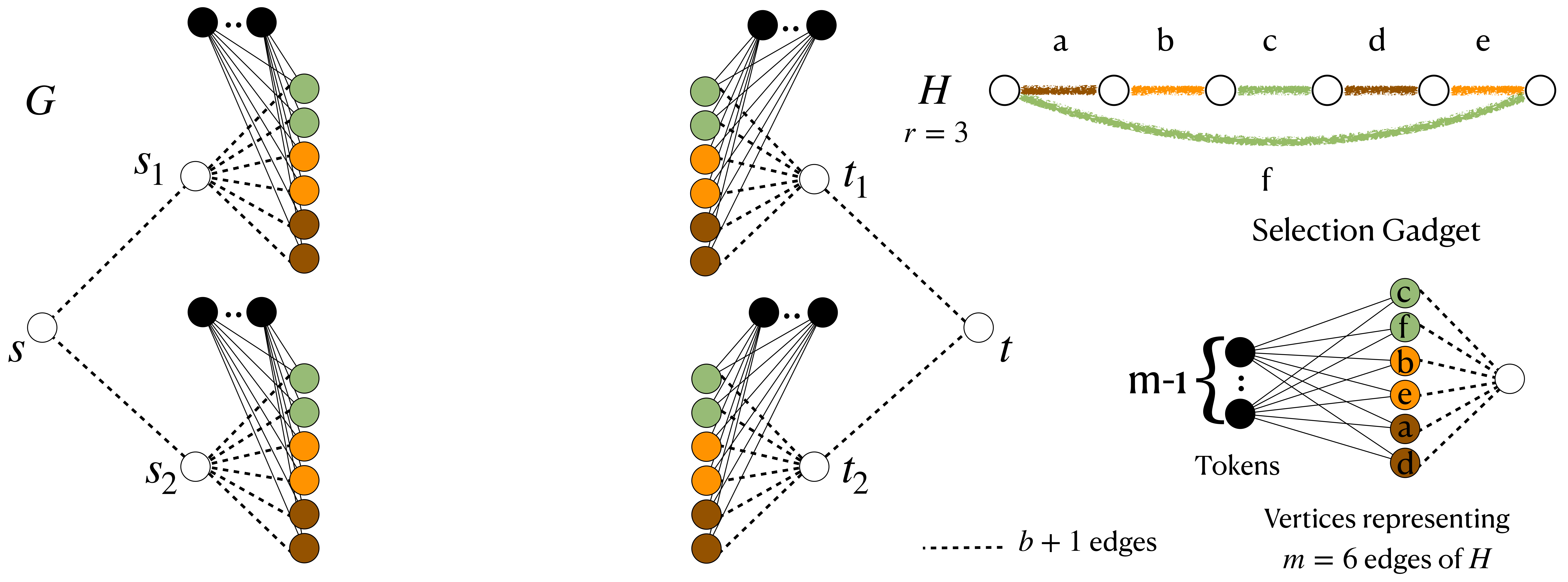

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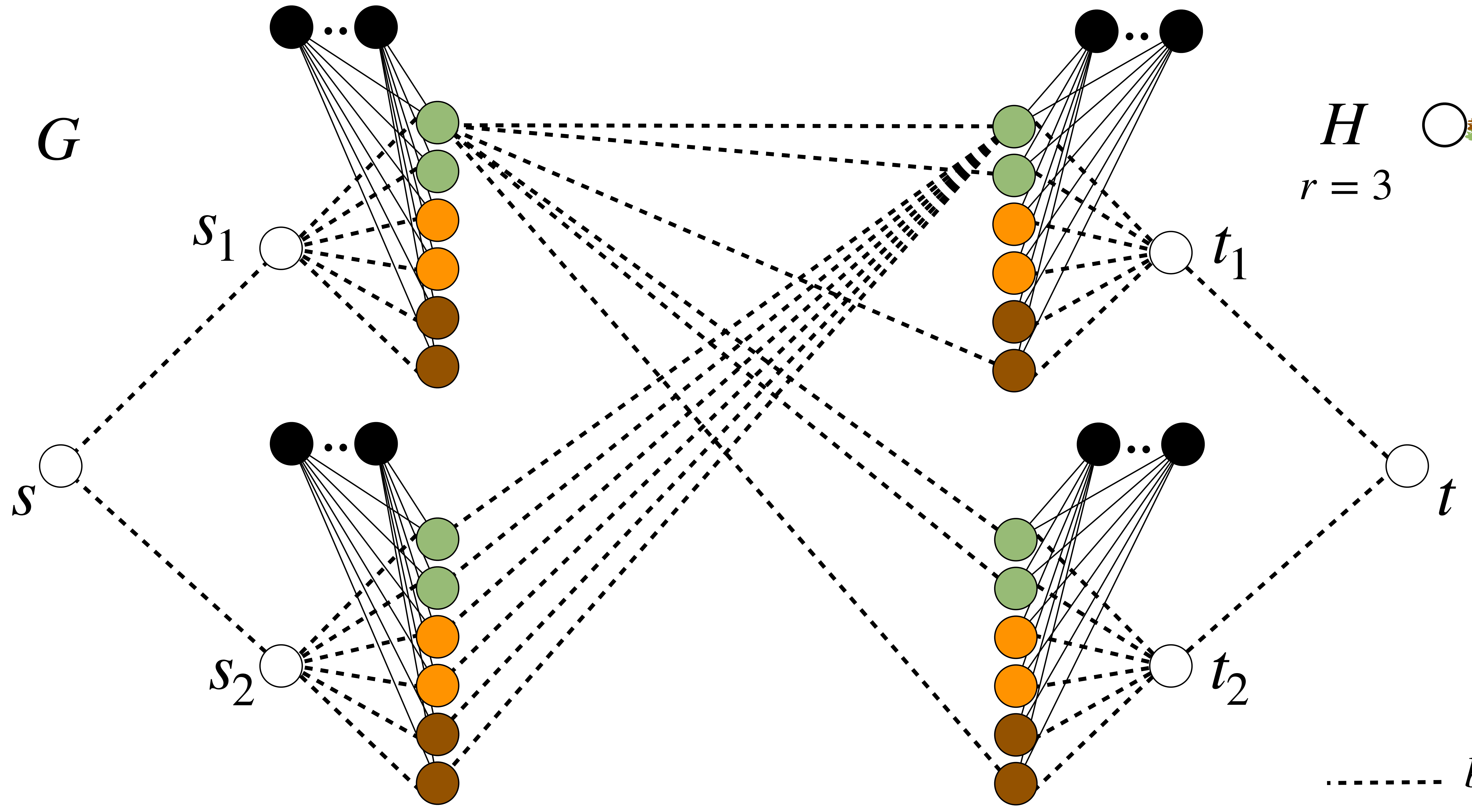

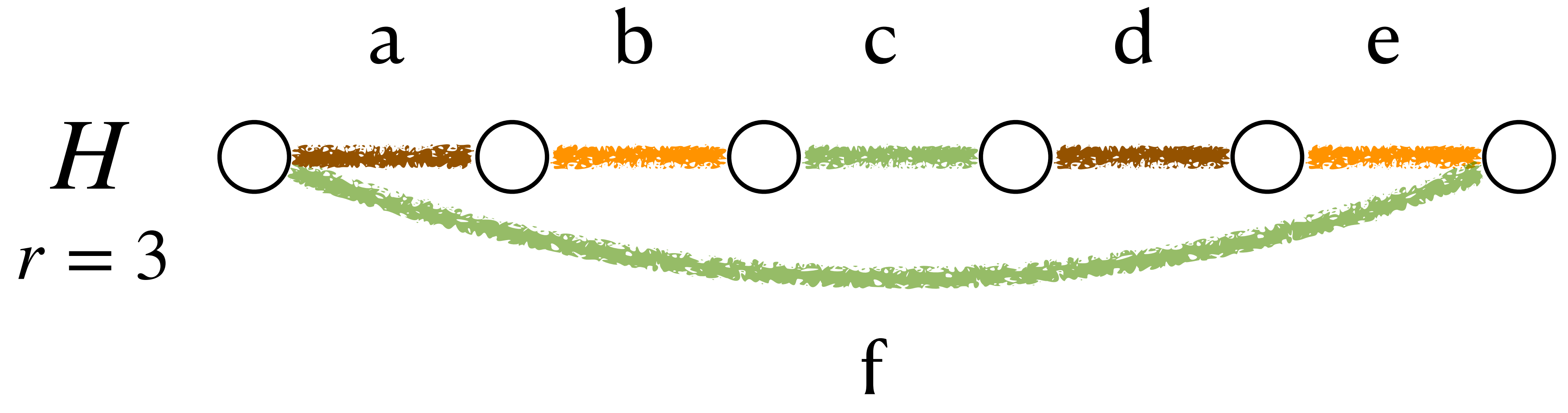

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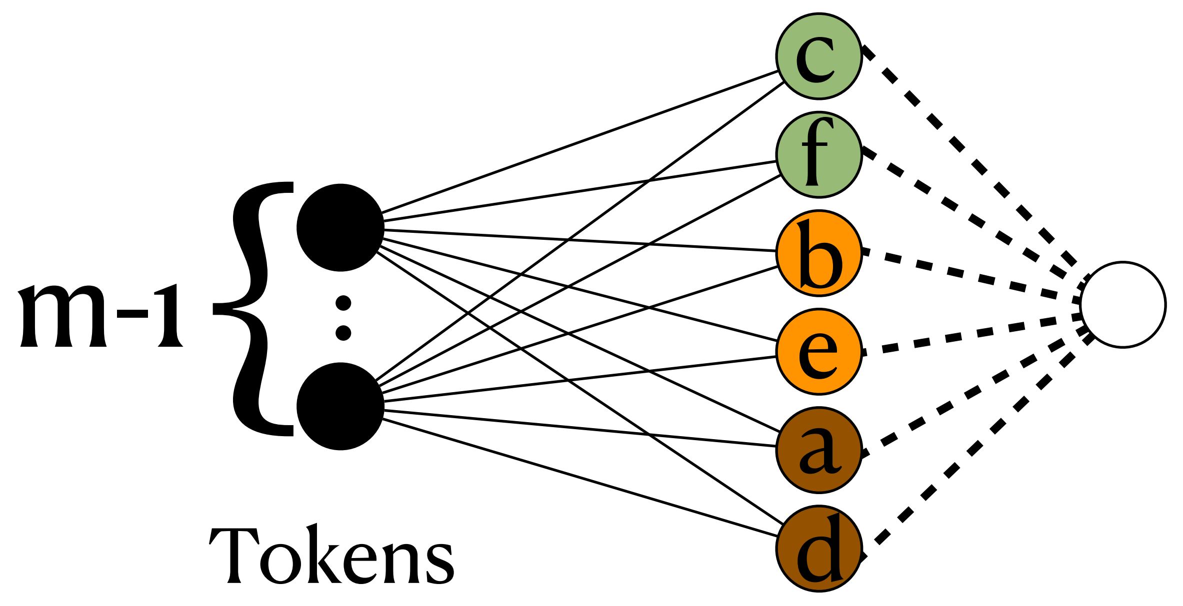

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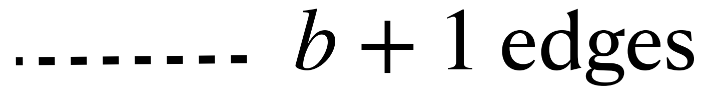

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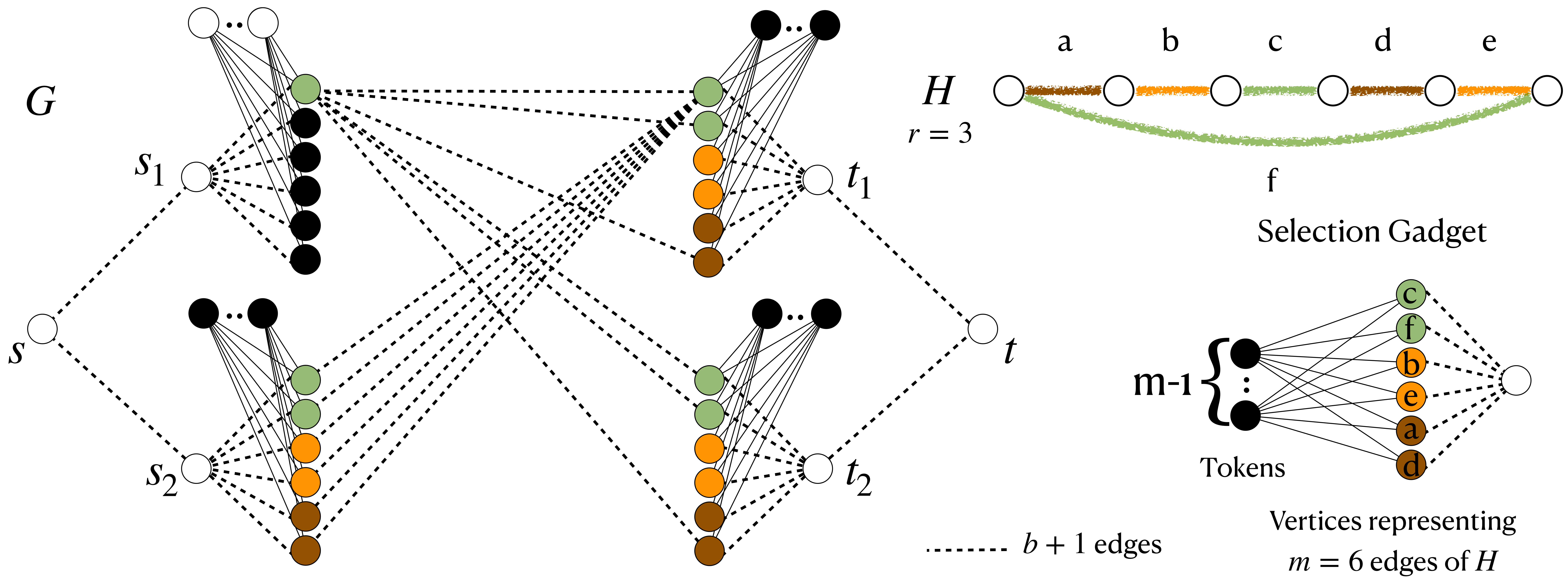

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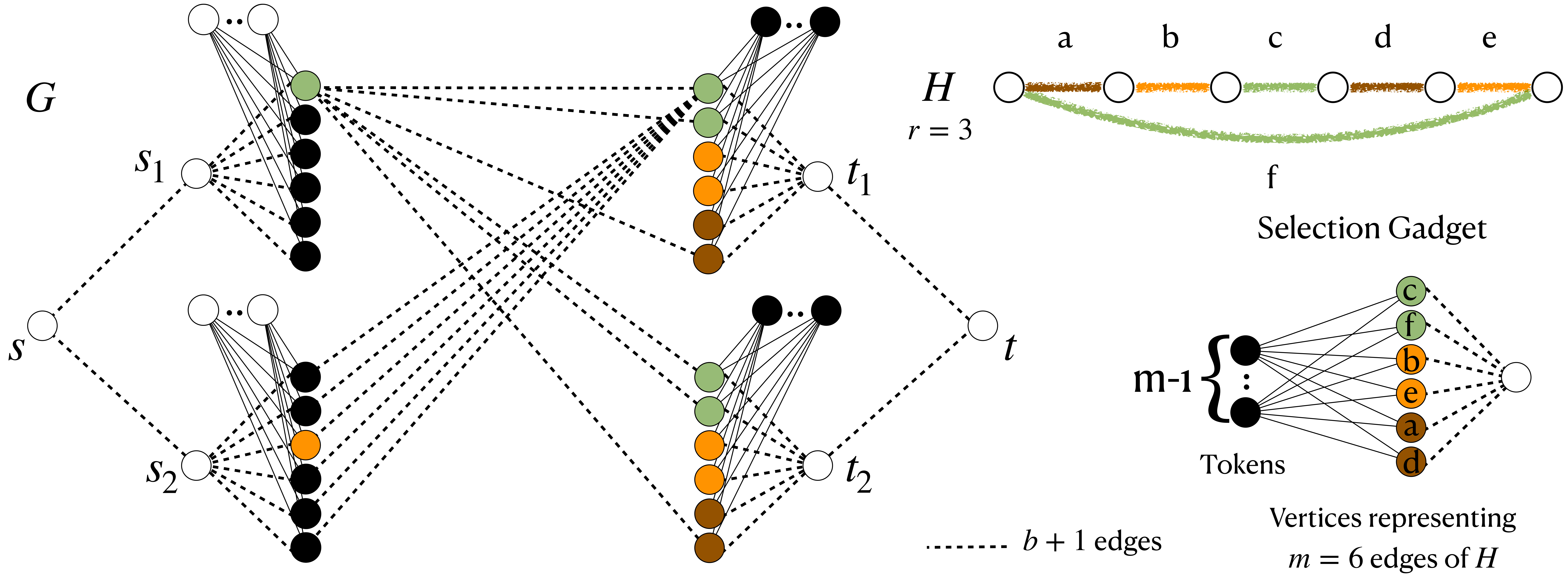

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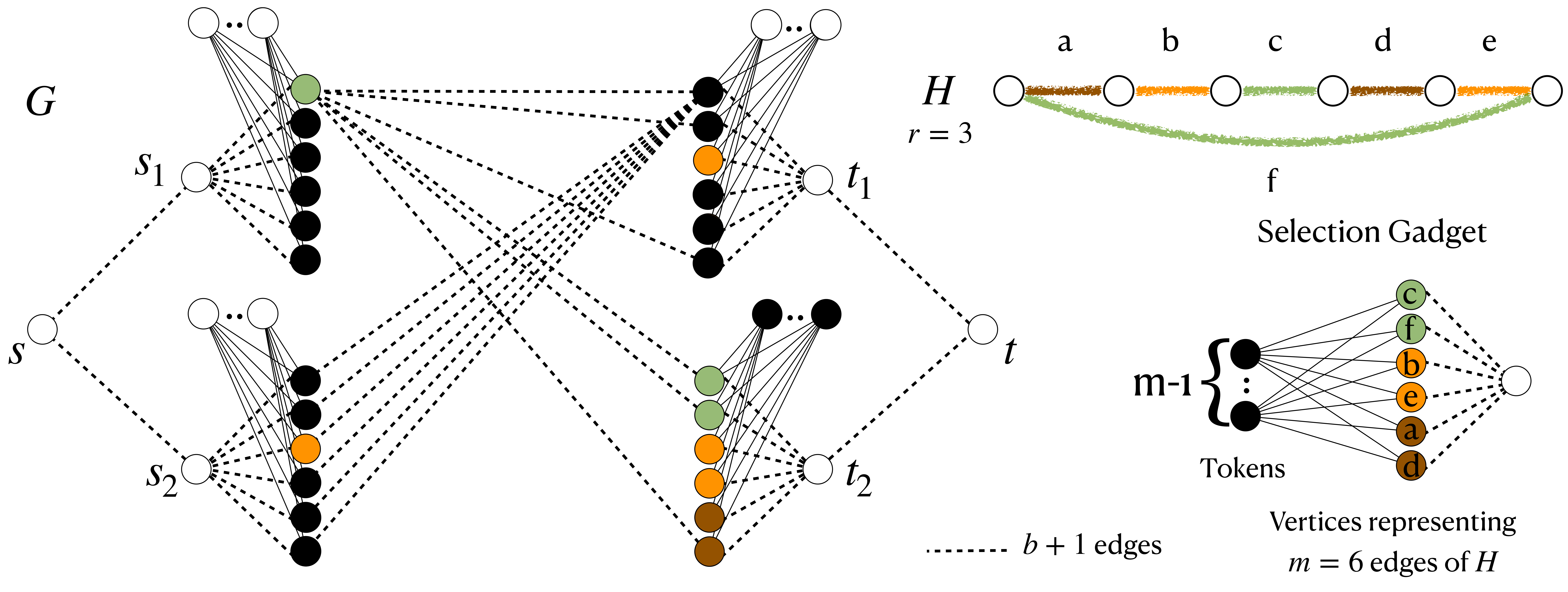

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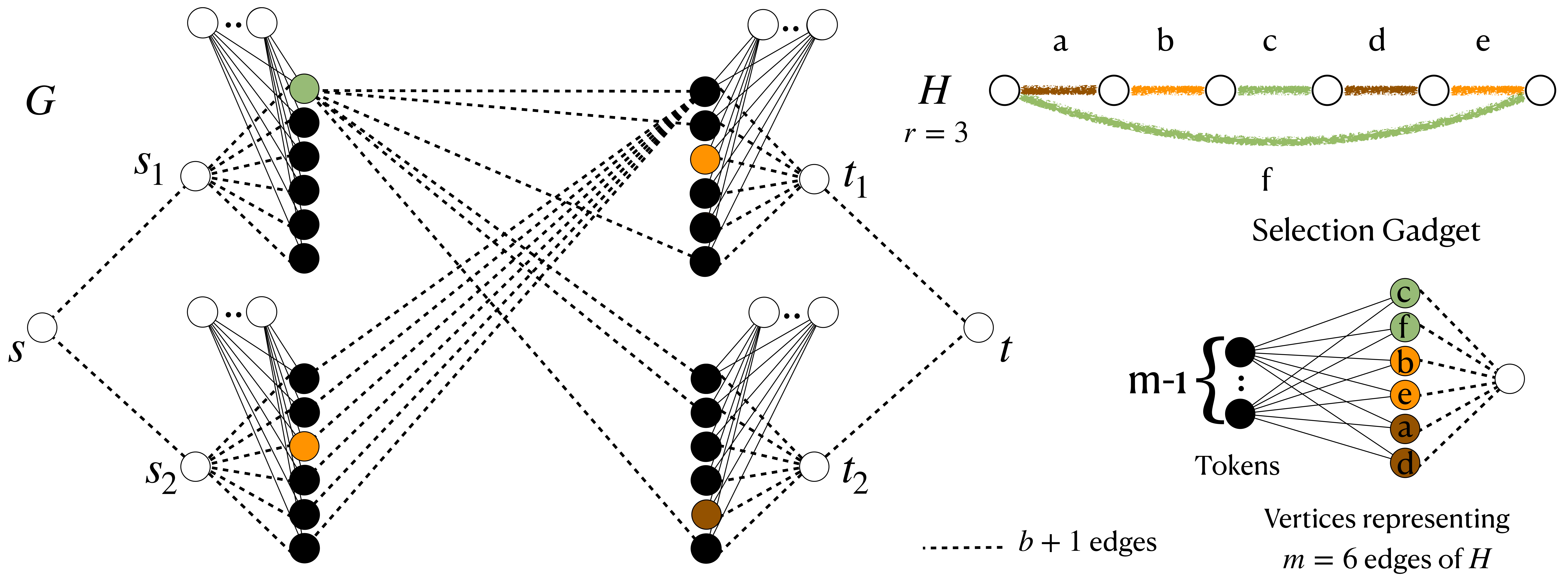

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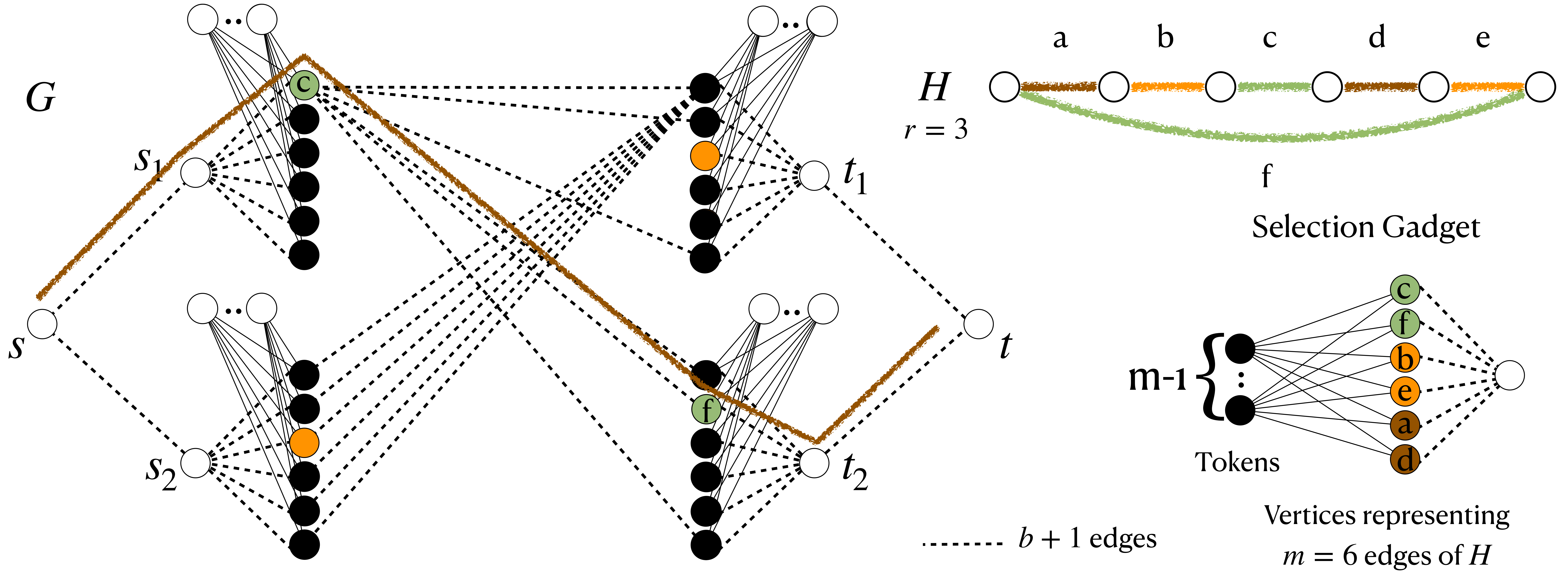

- If a cut can be reached in fewer than b token slides, the vertices that remain free in the gadgets represent the edges of the rainbow matching (and vice versa).

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Naomi Nishimura

Mario Grobler

Thank you.

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